

# HYDROLOGICAL RECIPES

ESTIMATION TECHNIQUES IN  
AUSTRALIAN HYDROLOGY



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# ERRATA FOR HYDROLOGICAL RECIPES

## MAY 2004

*The following errors have been discovered to date. We are thankful to those who have made us aware of their existence. Please notify the CRC for Catchment Hydrology (crcch@eng.monash.edu.au) if you are aware of other errors.*

- Page 16** In the definition of terms for equation 4.2.7, the reference for the computation of N (total day length) should refer to equation 4.2.3.
- Page 16** In equation 4.2.9,  $e_a$  should read  $e_d$ , the actual vapour pressure [kPa] – see equation 4.6.3.
- Page 17** Example Last line,  $R_{nl} = 3.88 \text{ MJm}^{-2}\text{d}^{-1}$  (Small difference, therefore no change to  $R_n$ )
- Page 19** Section 4.3 has now been superseded by similar analysis over the whole of Australia, published by the Bureau of Meteorology as “Climatic Atlas of Australia: Evapotranspiration”.
- Page 33** The psychrometric constant  $\gamma$  should be 0.066.
- Page 34** An additional method for computing  $e_d$  from climate data is to use pairs of wet and dry bulb temperature (i.e. not requiring RH data) using:

$$e_d = 0.611 \exp\left(\frac{17.27T_{\text{wet}}}{T_{\text{wet}} + 237.3}\right) - \gamma[T_{\text{dry}} - T_{\text{wet}}]$$

Where  $\gamma$  is the psychrometric constant ( $\approx 0.066$ ).

As many pairs of  $T_{\text{dry}}$  and  $T_{\text{wet}}$  as are available from throughout the 24 hour period should be used to calculate a range of  $e_d$  values, which can then be averaged to give a daily value.

However, because  $e_d$  does not vary a lot over a day (compared to other variables), if only one pair of  $T_{\text{dry}}$  and  $T_{\text{wet}}$  is available, it may be used as a reasonable estimate for  $e_d$  for the day.

- Page 34** The constant 409.8 in equation 4.6.4 should be 4098, and the denominator should be squared. Therefore Equation 4.6.4 should read:

$$\Delta = \frac{4098e_a}{(T + 237.3)^2}$$

- Page 35** In the example,  $\gamma$  should be 0.066, and  $\Delta$  should be  $0.108 \text{ kPaC}^{-1}$ . These changes do not affect the answer because, in this case,  $ET_0$  is not sensitive to  $\gamma$  and  $\Delta$ .
- Page 62** Equation 5.6.3 should read  $\text{Var}(r_1) = (n^3 - 3n^2 + 4) / [n^2(n^2 - 1)]$ . The z-statistic for the Autocorrelation test in the example on page 69 should therefore have a value of 1.523. The conclusion remains the same.
- Page 63** Note that equation 5.6.13 is correct, but the equivalent equation in Chiew and McMahon 1993 is wrong.
- Page 70** The K-W statistic of 9.21 is statistically significant at the 10% level but not the 5% level.
- Page 103** In Table 7.4.1, the exponent in the equation for  $q_{\text{smaf}}$  should be +1.0202 and in the line above,  $q_{\text{maf}}$  should read  $q_{\text{smaf}}$
- Page 106** In the example,  $S = 9.577$  million  $\text{m}^3$  or approximately 10,000 ML.

# HYDROLOGICAL RECIPES

## ESTIMATION TECHNIQUES IN AUSTRALIAN HYDROLOGY

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## **PREFACE**

The main activity of the CRC for Catchment Hydrology is cooperative research. With this cooperation taking place between all of its industry and research parties, the CRC is well placed to produce outcomes from its core research projects that meet industry needs and lead to improved catchment and land management. For its major projects, the strategy is to create new knowledge in priority research areas, and use the interaction between industry and research available in the CRC to facilitate its adoption.

“Hydrological Recipes” is an example of another kind of cooperative activity encouraged by the CRC structure. It has involved both industry and research staff in identifying the needs of industry that could be met (primarily) from existing knowledge. Here we are talking about procedures that were not well known, were in a form that wasn’t easy to understand, or simply had been developed but not documented. Doubtless, there are plenty of other recipes to add to this initial batch, so an update or sequel is contemplated.

Many people have contributed to this first edition of “Hydrologic Recipes”; without their knowledge, ideas, enthusiasm, and energy, we couldn’t have made the concept a reality. The goal of putting ideas in a form useful to the profession and sharing them is one which has been a feature of the short history of modern hydrology; the production of “Recipes” shows that we plan to continue this practice in the CRC for Catchment Hydrology.

John Langford  
Chairman

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## 1.1 INTRODUCTION

### *Aim of this handbook*

The aim of this handbook is to bring together a group of “quick” methods (or “recipes”) for hydrological analyses, aimed particularly for Australian conditions. The methods presented have proved useful to their developers, but until now, have not been readily accessible to the hydrological practitioner.

### *Scope*

The handbook begins with an outline of Australian streamflow in comparison with streamflows in other continents. It is followed by topics on areal reduction factors for precipitation and various methods for the computation of evaporation and evapotranspiration. The next two major sections relate to the analysis of flow and storage yield information including methods for analysing trends in data. A number of methods are presented for estimating hydrological characteristics in ungauged areas, followed by topics related to the river environment.

The topics covered by the “recipes” in this handbook are by no means complete. Our aim was to incorporate a range of information so that the usefulness of the concept could be assessed and to generate feedback on possible topics for a second volume.

### *Users*

It is envisaged that the handbook will be used by people with a background in hydrology. Readers without hydrological knowledge are cautioned against using the methods in this handbook without first seeking advice from a hydrologist.

### *Format of topics*

Each topic is described without recourse to detailed theoretical development and there is little background material presented. Examples are included where suitable. There is also some guidance provided as to appropriate applications but these caveats do not diminish the individual's responsibility for their final decision whether or not to use the technique. Direct references and suggested reading lists should be studied in detail if one is unsure about the applicability of a particular method. The topics have been reviewed by researchers and practitioners, as appropriate, to ensure the technical information is correct.

### *Future topics - An invitation*

There is a bias towards South Eastern Australia in some topics, due to the ready availability of information. A second Volume of new topics is planned for 1999 and we welcome input from all States and Territories.

The mechanism for additional topics to be considered is to send appropriate material to the CRC for Catchment Hydrology marked "Hydrological Recipes". This will then be formatted and returned to the sender for checking. The checked topic will be reviewed in the same manner as existing topics. Topics will be accepted and incorporated into the next volume following successful reviews. Suggestions for future topics may also be sent to the CRC for Catchment Hydrology.

Note: Regular reference is made throughout the handbook to ARR87. This is "Australian Rainfall and Runoff - A guide to flood estimation". It is the standard book of methods related to flood estimation practice used by Australian engineers.

### *Reference*

Institution of Engineers Australia, 1987: *Australian Rainfall and Runoff - A guide to flood estimation*. Institution of Engineers, Australia, 374pp.

## 2.1 AUSTRALIAN STREAMFLOW - GLOBAL COMPARISONS

In recent years, a great deal of work has been undertaken to prepare and analyse hydrological records from stations around the world in order to search for different responses at the continental scale. These analyses have concluded that Australian and South African catchments respond quite differently (in terms of comparative statistics) to those from the rest of the world. The following tables are reproduced from McMahon et al. (1992) in order for readers to gain a broad appreciation of the behaviour of stream flow in different parts of the world and to provide a basis for comparison with catchments of interest.

### *Definitions for Tables that follow*

$C_v$	coefficient of variation = standard deviation/mean
$V_{\max}/V_{\text{mean}}$	ratio of maximum observed annual flow volume to average mean annual flow volume.
$t_{80}$	storage capacity required to meet 80% of draft at 95% reliability divided by the mean annual flow. This is based on the Gould Gamma procedure in Topic 5.3.
$q_s$	peak discharge expressed as a ratio of catchment area (specific discharge) [ $\text{m}^3 \text{s}^{-1} \text{km}^{-2}$ ]
$I_v$	the standard deviation of logarithms of the series of annual peak discharge [ $\text{m}^3 \text{s}^{-1}$ ]. This is used in fluvial geomorphology as a "flash flood index" (Baker, 1977).
$q_{100}/q_{\text{mean}}$	ratio of 1:100 year annual peak discharge to average annual peak discharge
MAR	mean annual runoff [mm]

### *References*

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### *Further Reading*

- Falkenmark, M and T. Chapman Eds. (1989): *Comparative hydrology - an ecological approach to land and water resources*. UNESCO, Paris, 479pp.

Table 2.1 Some hydrologic parameters based on annual streamflow volumes

Note: The 'All' category in this table does not give a measure of the mean value for the continent, only of the data used in this study.

Continent	Area ranges (km <sup>2</sup> )	No. of streams	Average record length (years)	Average catchment area (km <sup>2</sup> )	Average MAR (mm)	Average C <sub>v</sub>	Average V <sub>max</sub> /V <sub>mean</sub>	Average t <sub>80</sub>
WORLD	0-10 <sup>3</sup>	434	28	322	820	0.45	2.2	0.79
	10 <sup>3</sup> -10 <sup>4</sup>	273	35	3376	540	0.48	2.5	0.99
	10 <sup>4</sup> -10 <sup>5</sup>	180	36	37675	410	0.37	2.0	0.54
	> 10 <sup>5</sup>	87	47	527312	230	0.33	2.0	0.46
	All	974	33	55153	610	0.43	2.2	0.77
NORTHERN AFRICA	0-10 <sup>3</sup>	0						
	10 <sup>3</sup> -10 <sup>4</sup>	3	11	3836	170	0.54	2.2	0.86
	10 <sup>4</sup> -10 <sup>5</sup>	5	34	48168	190	0.37	2.0	0.39
	> 10 <sup>5</sup>	15	30	301913	210	0.25	1.6	0.23
	All	23	29	207872	200	0.31	1.8	0.35
SOUTHERN AFRICA	0-10 <sup>3</sup>	55	26	298	280	0.81	3.6	2.38
	10 <sup>3</sup> -10 <sup>4</sup>	30	31	3315	100	0.78	3.5	1.82
	10 <sup>4</sup> -10 <sup>5</sup>	11	30	31235	180	0.70	3.2	1.61
	> 10 <sup>5</sup>	4	38	299254	70	0.54	2.5	0.98
	All	100	28	16565	210	0.78	3.5	2.07
ASIA	0-10 <sup>3</sup>	42	16	242	900	0.47	2.1	0.81
	10 <sup>3</sup> -10 <sup>4</sup>	31	21	4637	790	0.45	2.4	1.06
	10 <sup>4</sup> -10 <sup>5</sup>	45	14	32163	400	0.30	1.6	0.31
	> 10 <sup>5</sup>	25	36	734462	310	0.28	1.7	0.29
	All	143	20	139600	620	0.38	2.0	0.62
NORTH AMERICA	0-10 <sup>3</sup>	83	23	347	1690	0.31	1.7	0.30
	10 <sup>3</sup> -10 <sup>4</sup>	54	47	3142	720	0.39	2.2	0.54
	10 <sup>4</sup> -10 <sup>5</sup>	33	45	36491	510	0.38	2.2	0.63
	> 10 <sup>5</sup>	19	61	403496	150	0.35	2.1	0.42
	All	189	37	47984	1050	0.35	2.0	0.44
SOUTH AMERICA	0-10 <sup>3</sup>	11	34	516	640	0.39	2.2	0.47
	10 <sup>3</sup> -10 <sup>4</sup>	17	34	3538	730	0.33	1.9	0.34
	10 <sup>4</sup> -10 <sup>5</sup>	21	38	39614	670	0.34	2.0	0.42
	> 10 <sup>5</sup>	4	47	1983646	450	0.41	3.7	0.96
	All	53	36	166647	670	0.35	2.1	0.44
EUROPE	0-10 <sup>3</sup>	103	34	334	520	0.30	1.7	0.28
	10 <sup>3</sup> -10 <sup>4</sup>	79	33	3234	510	0.27	1.6	0.24
	10 <sup>4</sup> -10 <sup>5</sup>	61	47	42500	350	0.31	1.8	0.35
	> 10 <sup>5</sup>	17	65	342758	250	0.25	1.8	0.19
	All	260	39	33497	460	0.29	1.7	0.28
SOUTH PACIFIC	0-10 <sup>3</sup>	40	20	299	1170	0.26	1.5	0.24
	10 <sup>3</sup> -10 <sup>4</sup>	10	18	2433	1750	0.22	1.4	0.15
	10 <sup>4</sup> -10 <sup>5</sup>	0						
	> 10 <sup>5</sup>	0						
	All	50	20	725	1290	0.25	1.5	0.22
AUSTRALIA	0-10 <sup>3</sup>	100	35	323	540	0.59	2.6	1.10
	10 <sup>3</sup> -10 <sup>4</sup>	49	42	3213	210	0.88	4.0	2.53
	10 <sup>4</sup> -10 <sup>5</sup>	4	46	30275	81	0.98	4.3	3.09
	> 10 <sup>5</sup>	3	30	120333	37	1.12	4.8	3.50
	All	156	37	4307	420	0.70	3.1	1.65

Table 2.2 Some hydrologic parameters based on annual peak discharges

Note: The 'All' category in this table does not give a measure of the mean value for the continent, only of the data used in this study.

Continent	Area ranges (km <sup>2</sup> )	No. of streams	Average record length (years)	Average catchment area (km <sup>2</sup> )	Average $\bar{q}_s$ (m <sup>3</sup> /s/km <sup>2</sup> )	Average I <sub>v</sub>	Average Q <sub>100Q<sub>mean</sub></sub>
WORLD	0-10 <sup>3</sup>	423	27	302	0.80	0.32	4.5
	10 <sup>3</sup> -10 <sup>4</sup>	244	32	3535	0.23	0.29	5.1
	10 <sup>4</sup> -10 <sup>5</sup>	176	31	36271	0.08	0.25	3.9
	> 10 <sup>5</sup>	88	42	548994	0.03	0.17	2.9
	All	931	30	59812	0.44	0.28	4.4
NORTHERN AFRICA	0-10 <sup>3</sup>	0					
	10 <sup>3</sup> -10 <sup>4</sup>	7	15	3721	0.12	0.34	16.8
	10 <sup>4</sup> -10 <sup>5</sup>	10	30	34065	0.04	0.23	2.8
	> 10 <sup>5</sup>	15	35	566690	0.02	0.08	1.4
	All	32	29	277095	0.05	0.18	5.2
SOUTHERN AFRICA	0-10 <sup>3</sup>	72	26	210	0.46	0.60	5.6
	10 <sup>3</sup> -10 <sup>4</sup>	25	31	3482	0.15	0.47	13.6
	10 <sup>4</sup> -10 <sup>5</sup>	10	29	28250	0.08	0.39	12.4
	> 10 <sup>5</sup>	4	27	152020	0.01	0.40	5.7
	All	111	27	8944	0.34	0.54	8.1
ASIA	0-10 <sup>3</sup>	40	16	262	0.58	0.35	7.8
	10 <sup>3</sup> -10 <sup>4</sup>	31	19	4111	0.40	0.23	3.2
	10 <sup>4</sup> -10 <sup>5</sup>	43	14	31832	0.11	0.22	3.3
	> 10 <sup>5</sup>	24	42	691434	0.04	0.16	3.4
	All	138	21	131168	0.30	0.25	4.6
NORTH AMERICA	0-10 <sup>3</sup>	111	24	266	1.36	0.26	4.0
	10 <sup>3</sup> -10 <sup>4</sup>	37	34	4182	0.30	0.27	4.6
	10 <sup>4</sup> -10 <sup>5</sup>	28	32	32506	0.08	0.27	3.9
	> 10 <sup>5</sup>	19	39	482158	0.02	0.16	2.2
	All	195	28	52592	0.85	0.25	3.9
SOUTH AMERICA	0-10 <sup>3</sup>	2	17	808	0.53	0.18	5.7
	10 <sup>3</sup> -10 <sup>4</sup>	6	25	3772	0.23	0.16	2.1
	10 <sup>4</sup> -10 <sup>5</sup>	13	29	46768	0.11	0.14	2.0
	> 10 <sup>5</sup>	4	48	1557193	0.04	0.10	1.6
	All	25	30	274440	0.16	0.14	2.2
EUROPE	0-10 <sup>3</sup>	61	36	438	0.21	0.16	2.2
	10 <sup>3</sup> -10 <sup>4</sup>	76	36	3302	0.12	0.17	2.4
	10 <sup>4</sup> -10 <sup>5</sup>	56	41	42857	0.06	0.18	2.8
	> 10 <sup>5</sup>	16	57	363555	0.03	0.14	2.1
	All	209	39	40644	0.12	0.17	2.4
SOUTH PACIFIC	0-10 <sup>3</sup>	40	23	296	1.32	0.24	3.0
	10 <sup>3</sup> -10 <sup>4</sup>	12	22	2284	0.86	0.16	2.7
	10 <sup>4</sup> -10 <sup>5</sup>	0					
	> 10 <sup>5</sup>	0					
	All	52	23	755	1.21	0.22	2.9
AUSTRALIA	0-10 <sup>3</sup>	97	32	333	0.68	0.39	4.9
	10 <sup>3</sup> -10 <sup>4</sup>	50	40	3323	0.16	0.51	5.8
	10 <sup>4</sup> -10 <sup>5</sup>	16	42	29597	0.05	0.56	6.5
	> 10 <sup>5</sup>	6	41	233668	0.01	0.50	7.8
	All	169	36	12272	0.45	0.45	5.4

## 3.1 AREAL REDUCTION FACTORS

### *Introduction*

Design rainfall information for flood estimation is generally made available to designers in the form of *point rainfall intensities* (eg. the rainfall intensity-frequency-duration information given in ARR87). However, most flood estimates are required for catchments of significant size and will thus require a design estimate of the *areal average rainfall intensity* over the catchment. The ratio between the design values of areal average rainfall and point rainfall, computed for the same duration and average recurrence interval (ARI), is called the *areal reduction factor (ARF)*. It allows for the fact that larger catchments are less likely than smaller catchments to experience high intensity storms over the whole of the catchment area.

### *Derivation of Areal Reduction Factors for Victoria (18 to 120 hours)*

ARF values for a catchment of a given size can be determined from the analysis of rainfall data available at the gauges within that catchment. This requires separate frequency analysis of extreme values of point rainfall and areal rainfall for selected durations. The determination of average ARF values for a whole region requires the repetition of this procedure for many different catchments of that size.

The ARF values presented in this topic are based on a detailed study undertaken by the Cooperative Research Centre for Catchment Hydrology using daily rainfall data from over 2000 rain gauges in Victoria (Siriwardena and Weinmann, 1996). The methodology used is a modified version of Bell's method (Bell, 1976) and was selected on the basis of an extensive literature survey (Srikanthan, 1995).

Individual ARF values were computed for a large number of circular "sample catchments" of selected size, distributed over those parts of Victoria with a relatively high rain gauge density. The average *point rainfall frequency curve* for each sample was determined using a regional L-moment approach for the Generalised Extreme Value (GEV) distribution (Hosking, 1990). Average areal rainfalls for the catchment were computed using Thiessen weights, and the *areal rainfall frequency curve* was also fitted by the method of L-moments using a GEV distribution.

Sets of ARF values were computed for durations of 1, 2 and 3 days, catchment areas of 125, 250, 500, 1000, 2000, 4000 and 8000 km<sup>2</sup>, and for ARIs of 2, 5, 10, 20, 50 and 100 years. A single equation was then fitted to these results to represent the dependence of ARF values on rainfall duration, catchment area and rainfall frequency (or ARI).

$$ARF_{ARI} = 1.00 - 0.4(A^{0.14} - 0.7 \log_{10} D) D^{-0.48} + 0.0002(A)^{0.4} D^{0.41} \left( 0.3 + \log_{10} \left( \frac{1}{ARI} \right) \right) \quad (3.1)$$

where

- A catchment area [km<sup>2</sup>] - for areas of 1 to 10000 km<sup>2</sup>  
 D storm duration [hours] - for durations of 18 to 120 hours

The study identified small but statistically significant differences in ARF values for different parts of Victoria, probably reflecting differences in hydrometeorological factors such as typical storm sizes. However, there is at present insufficient information to allow differentiation of design values within Victoria based on catchment location. The application of a single set of design ARF values over the whole of Victoria is therefore recommended at this stage.

### Application

For the purpose of application, the general equation derived above was reformulated as an equation giving ARF values for a standard ARI of 2 years with a correction factor to calculate the ARF values for larger ARIs up to 100 years:

$$ARF_n = ARF_2 - K_n \quad (3.2)$$

where

- ARF<sub>n</sub> areal reduction factor for ARI of n years  
 ARF<sub>2</sub> areal reduction factor for ARI of 2 years  
 K<sub>n</sub> correction factor to convert from ARF<sub>2</sub> to ARF<sub>n</sub>

and

$$ARF_2 = 1.00 - 0.4(A^{0.14} - 0.7 \log_{10} D) D^{-0.48} \quad (3.3)$$

Figure 3.1 is a graphical representation of this equation and allows determination of ARF<sub>2</sub> (the ARF value for an ARI of 2 years) for durations in the range from 18 hours to 120 hours, and for catchment sizes from 1 km<sup>2</sup> to 10 000 km<sup>2</sup>. This ARF value can be used directly for design rainfalls in the range from 1 year to 2 years ARI.

A set of correction factors then allows the calculation of ARF values for larger ARIs, based on the following equation:

For catchment areas ≤ 10 km<sup>2</sup>

$$K_n = 0.00 \quad (3.4)$$

For catchment areas  $> 10 \text{ km}^2$

$$K_n = -0.0002(A)^{0.4} D^{0.41} \left( 0.3 + \log_{10} \left( \frac{1}{\text{ARI}} \right) \right) \quad (3.5)$$

### *Interim Areal Reduction Factors for Durations less than 18 hours*

ARF values for durations less than 18 hours are the subject of an ongoing study but an interim method is presented here.

The determination of reliable ARF values for durations less than 24 hours would need to be based on detailed analysis of pluviograph data for the region of interest. To date, there have not been any studies of sufficient regional extent and level of detail to provide a basis for firm recommendations on ARFs for short duration rainfalls in Victoria or other parts of Australia. In this situation, it is only possible to make recommendations on interim values of ARFs for short durations, based on published results of ARF studies for overseas regions and comparisons with the results of the limited studies undertaken for Australian regions.

The most comprehensive and relevant overseas studies are by the U.S. National Weather Service (1980), providing the basis for the currently adopted ARF values in Australia, and by the U.K. Institute of Hydrology, providing the basis for the ARF values in the U.K. Flood Studies Report (NERC, 1975). Comparisons of the 24-hour duration ARF values obtained by these overseas studies with the corresponding values from the Victorian study (Siriwardena and Weinmann, 1996) indicated closer agreement with U.K. values than with the U.S. values currently adopted in Australia. On this background, the ARF values for 1-hour duration presented in the U.K. Flood Studies Report (NERC, 1995) have been adopted as the basis for estimating short duration ARF values for S.E. Australia. The interim ARF values for duration between 1 hour and 18 hours were determined by assuming a linear variation with duration.

The adopted relationship for interim ARF values in the range of durations from 1 hour to 18 hours (applicable to all ARIs) can be expressed by the following equation:

$$\text{ARF} = 1.00 - 0.10(A^{0.14} - 0.879) - 0.029(A)^{0.233} (1.255 - \log_{10} D) \quad (4)$$

These **interim design values** are represented in Figure 3.2.

Note: While the ARF values presented here were derived for Victoria, they may be applied to other regions with similar hydrometeorological characteristics. There is work presently in progress (J.L. Irish, pers. comm.) to derive ARFs for parts of New South Wales and Southern Queensland. Comparison with results of the most relevant

Australian study (Masters and Irish, 1994, for Sydney area) indicated that the results so obtained are probably an adequate approximation for South East Australian conditions, being conservatively high for the Sydney area.

### Example

Determine the ARF value for a duration of 30 hours, a catchment size of 1600 km<sup>2</sup> and an ARI of 100 years.

The value of ARF<sub>2</sub> read from Figure 3.1 is 0.86. Equation 3.5 gives a K<sub>100</sub> for a duration of 30 hours of 0.03. Therefore:

$$\text{ARF}_{100} = 0.86 - 0.03 = 0.83.$$

### *References*

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- Siriwardena, L. and P.E. Weinmann (1996): *Areal reduction factors for design rainfalls in Victoria*. Cooperative Research Centre for Catchment Hydrology, (in preparation).
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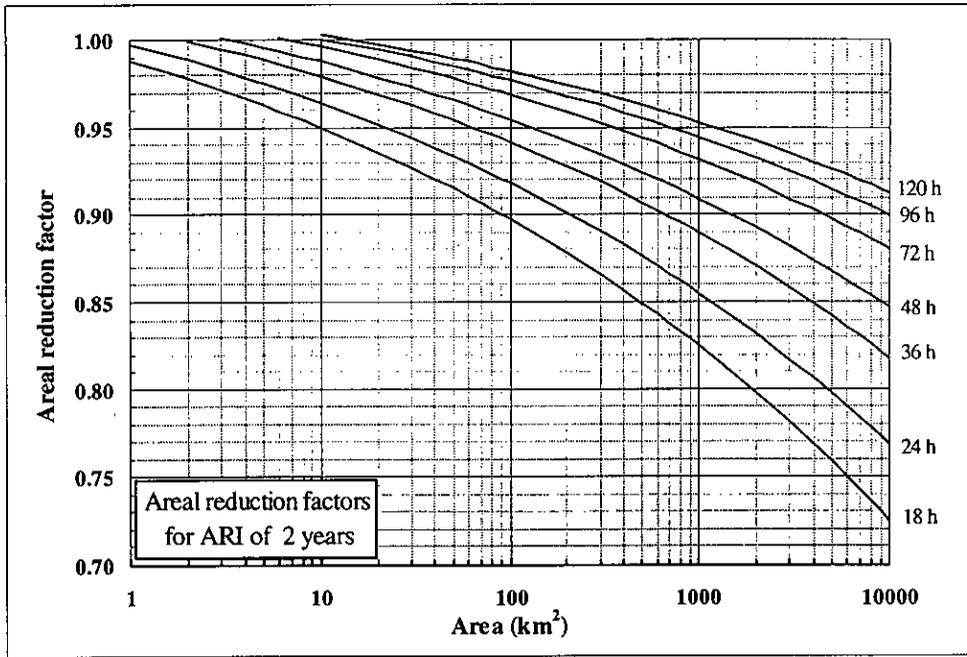


Figure 3.1 Arel reduction factors for Victoria - 18 to 120 hours duration

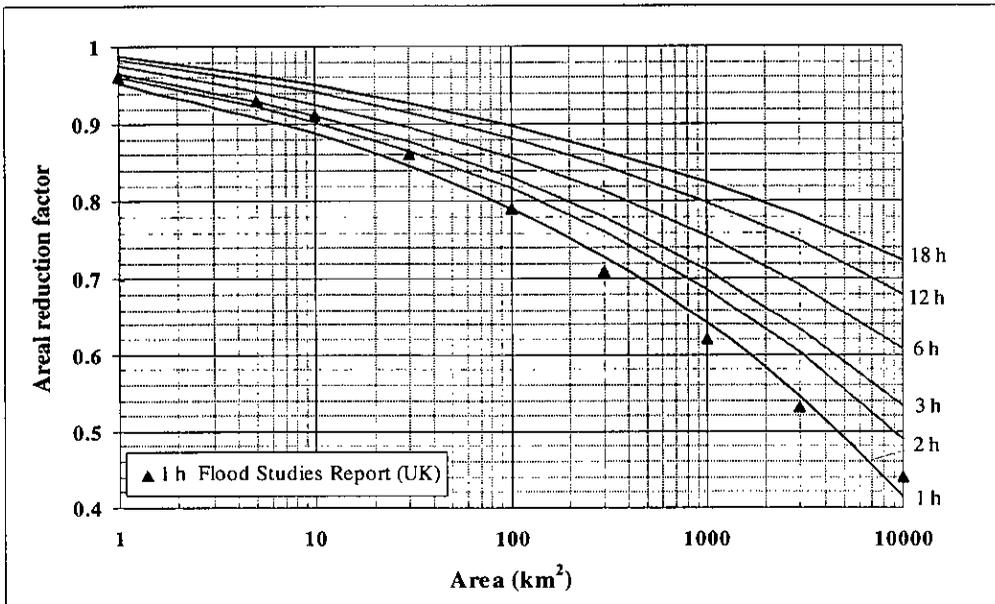


Figure 3.2 Interim arel reduction factors for Victoria - 1 to 18 hours duration



## 4.1 INTRODUCTION TO TOPICS ON EVAPORATION AND EVAPOTRANSPIRATION

This handbook contains several topics on estimating evaporation and evapotranspiration. This general area is of great importance to hydrology, especially in dry continents like Australia where evaporative losses make up major parts of the surface water balance. As an introduction to these topics, a series of definitions and descriptions follow to ensure readers are clear on the meaning of terms that are often rather loosely used.

**Evaporation** is a term used to describe the amount of water that passes or could pass into the atmosphere across a soil/air, water/air or plant/air interface.

**Evapotranspiration** is often used interchangeably with evaporation but is intended to stress the point that water can cross plant/air interfaces i.e. it is common to use "evaporation" when talking about open water surface and bare soil, but "evapotranspiration" when referring to land surfaces with plants. This is the way the terms are used in this handbook.

**Potential Evaporation/Evapotranspiration** is the maximum amount of water that can evaporate/transpire from a surface where water availability is not limiting (i.e. a well watered surface or open water body).

**Actual Evaporation/Evapotranspiration** is the actual amount of water that crosses an interface into the air. This is generally constrained by the atmospheric conditions during wet periods and by the soil moisture/plant physiology during dry periods.

**Reference crop evapotranspiration** ( $ET_0$ ) is formally defined as "the rate of evapotranspiration from a hypothetical crop with an assumed crop height (0.12 m) and a fixed canopy resistance ( $70 \text{ s m}^{-1}$ ) and albedo (0.23) which would closely resemble evapotranspiration from an extensive surface of green grass cover of uniform height, actively growing, completely shading the ground and not short of water" (Smith et al., 1992). This is commonly known as Penman-Monteith potential evapotranspiration for grass.

**Wet environment areal evapotranspiration** as defined by Morton (1983) is the evapotranspiration rate that would occur from a soil-plant surface that is saturated and has no limitations on water availability. This is similar to the definition of potential evapotranspiration adopted above but takes account of the feedback that evaporation may have on air temperature and humidity. It is a term used exclusively for evaporation estimates using Morton's method. Its computation is based on a modified version of the Priestly -Taylor equation (Priestly and Taylor, 1972).

**FAO-24 Radiation** is an estimation method for reference crop evapotranspiration ( $ET_0$ ) described in Topic 4.5 and recommended by the Food and Agriculture Organisation (1984). It is another modified version of the Priestly-Taylor equation (Priestly and Taylor, 1972).

**Surface albedo** is the fraction of incoming shortwave (solar) radiation that is reflected by a surface back into the atmosphere.

**Longwave radiation** refers to that of wavelengths greater than 4  $\mu\text{m}$ . Virtually all radiation emitted by the earth and atmosphere is in this range.

**Shortwave radiation** refers to that of wavelengths less than 4  $\mu\text{m}$ . Because virtually all radiation from the sun is in this range and the terms **shortwave** and **solar radiation** are often used interchangeably.

**Global radiation** is a term often used in meteorological measurement. It is the shortwave radiation received by the surface, consisting of both direct and diffuse solar radiation.

## *References*

- Food and Agriculture Organisation (1984): *Crop Water Requirements*. FAO Irrigation and Drainage Paper 24, 144pp.
- Morton, F.I. (1983): Operational estimates of areal evapotranspiration and their significance to the science and practice of hydrology. *Jour. of Hydrology*, 66, pp. 1-76.
- Priestly, C.H.B. and R.I. Taylor (1972): On the assessment of surface heat flux and evaporation using large scale parameters. *Monthly Weather Review*, 100(2), pp. 9-29.
- Smith, M. (Ed.), Allen, R., Monteith, J.L., Perrier, A., Santos Pereira, L. and A. Segeren (1992): *Expert Consultation on Revision of FAO Methodologies for Crop Water Requirements*. Land and Water Division, Food and Agriculture Organisation of the United Nations.

## *Further Reading*

- Allen, R.G. (1986): A Penman for all seasons. *Jour. Irrig. and Drainage*, ASCE, 112(4), pp. 348-368.
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- Jensen, M.E., Burman, R.D. and R.G. Allen (Eds.), (1990): *Evapotranspiration and irrigation water requirements*. Manual of Practice No. 70, ASCE, New York.

## 4.2 CALCULATION OF RADIATION

### *Net Radiation Flux from Sunshine Hours and Temperature*

In the calculation of evapotranspiration using approaches involving energy balance, it is necessary to obtain values of net radiation,  $R_n$ . This can be estimated from measured sunshine hours and mean daily temperature. The method presented here is recommended by an FAO expert panel (Smith et al., 1990) and can be considered a standard.

**NOTE: All angular calculations use radians.**

#### Solar Declination [rad]

$$\delta = 0.409 \sin(0.0172J - 1.39) \quad (4.2.1)$$

where

$$J \quad \text{Julian day [ 1 for 1 Jan.; 365 for 31 Dec.] for month M and day of the month, } D = \text{integer}(275 M/9 - 30 + D) - 2 \quad (4.2.2)$$

#### Total Daylength [hours]

The equation given here is based on the solar disc at zero elevation without refraction and may give slightly smaller values than tables based on time of sunset.

$$N = 7.64 \omega_s \quad (4.2.3)$$

where

$\omega_s$  sunset hour angle [rad]; given by

$$\omega_s = \arccos(-\tan\phi \tan\delta) \quad (4.2.4)$$

where

$\phi$  latitude [rad] (**negative for the Southern Hemisphere**)

$\delta$  solar declination [rad]

#### Extraterrestrial Radiation [ $\text{MJ m}^{-2} \text{d}^{-1}$ ]

This is the solar radiation incident at the top of the earth's atmosphere.

$$R_a = 37.6 d_r (\omega_s \sin\phi \sin\delta + \cos\phi \cos\delta \sin\omega_s) \quad (4.2.5)$$

where

$d_r$  relative distance between the Earth and Sun; given by:

$$d_r = 1 + 0.033 \cos(0.0172 J) \quad (4.2.6)$$

**Solar Radiation** [MJ m<sup>-2</sup> d<sup>-1</sup>]

$$R_s = \left( a_s + b_s \frac{n}{N} \right) R_a \quad (4.2.7)$$

where

- a<sub>s</sub> fraction of extraterrestrial radiation on overcast days ≈ 0.25 for average climate
- b<sub>s</sub> ≈ 0.50 for average climate (locally calibrated values of a<sub>s</sub> and b<sub>s</sub> are preferred)
- n sunshine hours per day [hours]
- N total day length from (3) [hours]
- R<sub>a</sub> extraterrestrial radiation [MJ m<sup>-2</sup> d<sup>-1</sup>]

Equation 4.2.7 gives quite good approximations to monthly totals but errors on a daily basis may be high.

If data on sunshine hours are not available, an approximation to the n/N ratio can be made from observations of cloudiness in oktas (Table 4.2.1). This is recommended only as a last resort.

Table 4.2.1 Indicative conversion from Cloudiness [oktas] to n/N ratio (after FAO, 1984)

Cloudiness [oktas]	0	1	2	3	4	5	6	7	8
n/N ratio	.95	.85	.75	.65	.55	.45	.35	.15	-

**Net Incoming Shortwave Radiation** [MJ m<sup>-2</sup> d<sup>-1</sup>]

$$R_{NS} = (1 - \alpha) R_s \approx 0.77 R_s \quad (4.2.8)$$

where

- α albedo ( 0.23 as an overall average for grass, 0.05 for a water surface)
- R<sub>s</sub> incoming solar radiation [MJ m<sup>-2</sup> d<sup>-1</sup>]

**Net Outgoing Longwave Radiation** [MJ m<sup>-2</sup> d<sup>-1</sup>]

$$R_{NL} = f \times 2.45 \times 10^{-9} [0.34 - 0.14\sqrt{e_a}] [T_{kx}^4 + T_{kn}^4] \quad (4.2.9)$$

where

- f adjustment for cloud cover = 0.1 + 0.9(n/N) (4.2.10)
- e<sub>a</sub> saturation vapour pressure [kPa] - see Topic 4.6, Equation 4.6.3
- T<sub>kx</sub> maximum day temperature [K]
- T<sub>kn</sub> minimum day temperature [K]

If no humidity data are available, the following approximation may be used:

$$R_{nl} \approx f [ 0.261 \exp[-7.77 \times 10^{-4} (T_k - 273)^2] - 0.02 ] \sigma T_k^4 \quad (4.2.11)$$

where

$\sigma$  Stefan-Boltzmann constant =  $4.9 \times 10^{-9}$  [MJ m<sup>-2</sup> K<sup>-4</sup> d<sup>-1</sup>]

$T_k$  mean air temperature [K]

### Net Radiation [MJ m<sup>-2</sup> d<sup>-1</sup>]

$$R_n = R_{ns} - R_{nl} \quad (4.2.12)$$

where

$R_{ns}$  net incoming shortwave radiation [MJ m<sup>-2</sup> d<sup>-1</sup>]

$R_{nl}$  net outgoing longwave radiation [MJ m<sup>-2</sup> d<sup>-1</sup>]

### Radiation Conversion

To convert between units used for radiation and evaporated water, use the following:

$$1 \text{ MJ m}^{-2} \text{ d}^{-1} = 11.57 \text{ Wm}^{-2} = 0.408 \text{ mm d}^{-1} \quad (\text{at } 20^\circ\text{C}) \quad (4.2.13)$$

There is a slight temperature dependency in this conversion but the value 0.408 may be used for the normal range of temperatures encountered with little error (Smith et al., 1992).

### Example

Estimate the net radiation at the Tatura for October 1 1989 given the following:  
latitude of 36.26 S (-0.633 rad - note that this is negative for the southern hemisphere),  
 $T_{\max}$ =16.3°C,  $T_{\min}$  = 11.8°C (i.e. mean temp = 14.1°C), 6.4 sunshine hours.

Julian day (4.2.2);	$J = 274$
Solar declination (4.2.1);	$\delta = -0.074$ rad
Sunset hour angle (4.2.4);	$\omega_s = 1.625$ rad
Total daylength (4.2.3);	$N = 12.42$ hours
Relative distance Sun - Earth (4.2.6);	$d_r = 1.00$
Extraterrestrial radiation (4.2.5);	$R_a = 32.87 \text{ MJ m}^{-2} \text{ d}^{-1}$
Solar radiation (4.2.7);	$R_s = 16.69 \text{ MJ m}^{-2} \text{ d}^{-1}$
Net shortwave radiation (4.2.8);	$R_{ns} = 12.85 \text{ MJ m}^{-2} \text{ d}^{-1}$
Net longwave radiation (4.2.9);	$R_{nl} = 3.82 \text{ MJ m}^{-2} \text{ d}^{-1}$

Net radiation (4.2.12);

$$R_n = 9.0 \text{ MJ m}^{-2} \text{ d}^{-1}$$

### *References*

Food and Agriculture Organisation (1984): *Crop Water Requirements*. FAO Irrigation and Drainage Paper **24**, 144pp.

Smith, M. (Ed.), Allen, R., Monteith, J.L., Perrier, A., Santos Pereira, L. and A. Segeren (1992): *Expert Consultation on Revision of FAO Methodologies for Crop Water Requirements*. Land and Water Division, Food and Agriculture Organization of the United Nations.

## 4.3 ESTIMATION OF MONTHLY WET ENVIRONMENT EVAPOTRANSPIRATION FOR VICTORIA

This handbook includes a number of methods for the determination of evapotranspiration. This topic presents one of the simplest methods for Victoria wherein the average value of wet environment areal evapotranspiration  $E_W$  for a particular site and month is read directly from a series of maps.

The maps were derived by Nathan and Pamminger (1995) using the method of Morton (1983) which requires wet and dry bulb temperatures and radiation input. Morton (1983) defines  $E_W$  as the evapotranspiration rate that would occur from a soil-plant surface that has no limitations on water availability (see Topic 4.1). For practical modelling purposes, Chiew and McMahon (1993) have shown that  $E_W$  is similar to the Penman-Monteith estimates of  $ET_0$ .

The data were available or could be reliably computed for 61 sites and 27 years of concurrent data. The computed values for each of the sites were then subject to a spline-type smoothing and interpolation method weighted by an elevation function.

A number of limitations on the use of the following maps should be recognised:

- (i) Analysis indicated some problems with estimates in north-west Victoria due most likely to problems in the Mildura radiation record.
- (ii) Although Nathan and Pamminger (1995) present maps of both wet environment areal and actual evapotranspiration, only  $E_W$  estimates are included in this handbook because of the likely problems with actual evapotranspiration estimates.
- (iii) The maps should be used to get estimates for catchments not specific points and errors are more likely near environmental discontinuities.
- (iv) While elevation was used in the smoothing and interpolation, there were no data from high altitude sites so estimates along the Great Dividing Range will be poor.

Maps of wet environment areal evapotranspiration are given for each month and annually on the following pages. Note that the maps are headed "POTENTIAL" but are calculated as Morton's wet environment areal evapotranspiration. These maps are reproduced with the permission of the Department of Natural Resources and Environment.

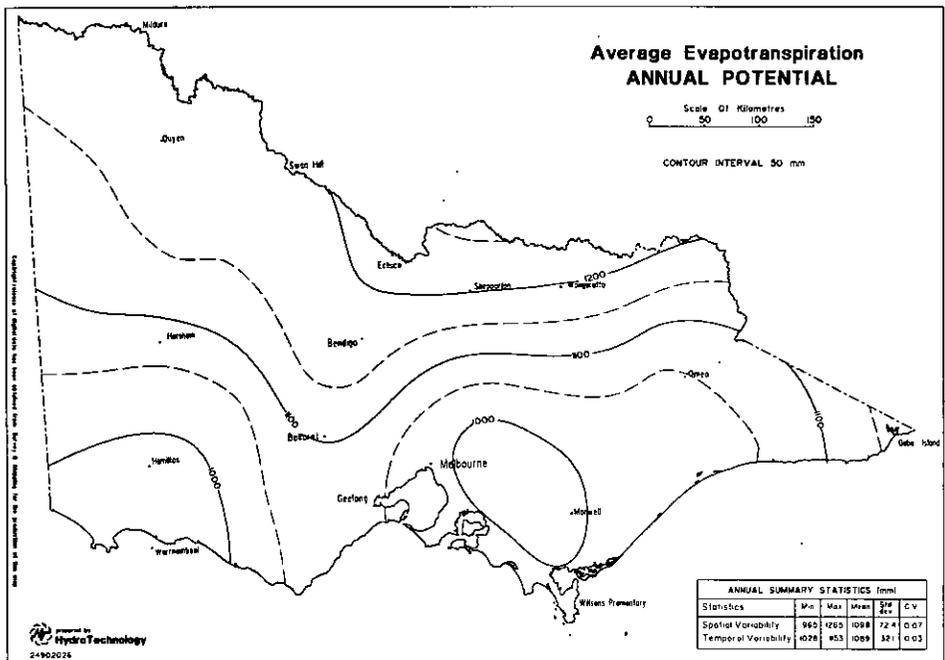
Note: A CRCCH project is presently in progress to produce similar maps for the whole of Australia by late 1997.

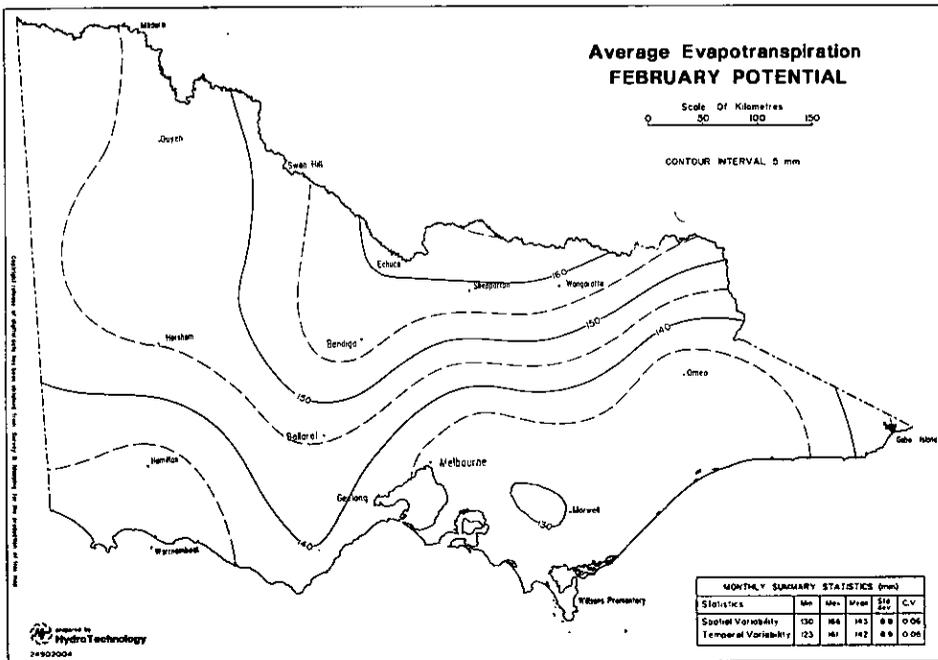
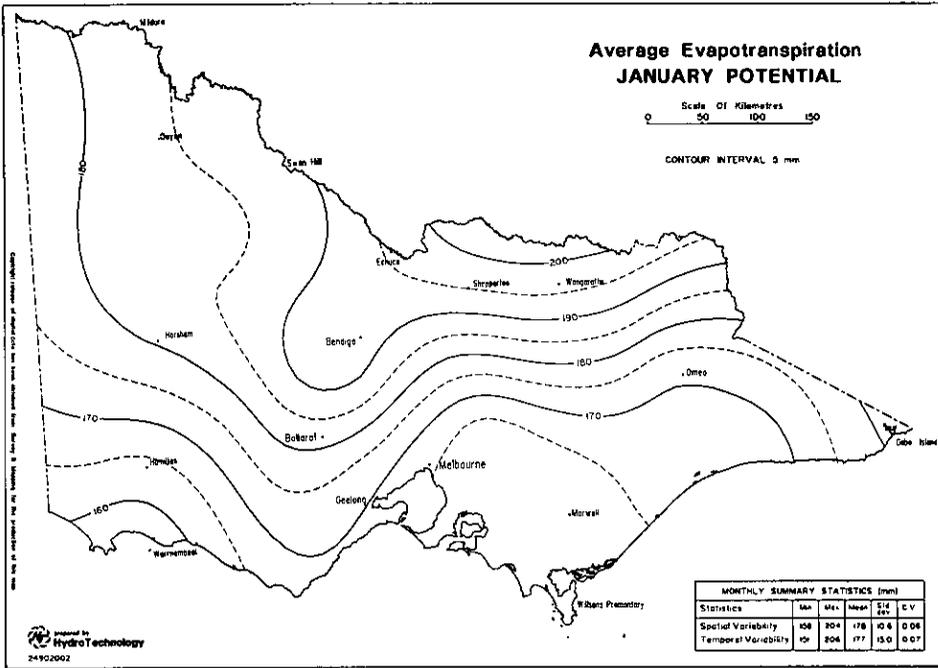
## References

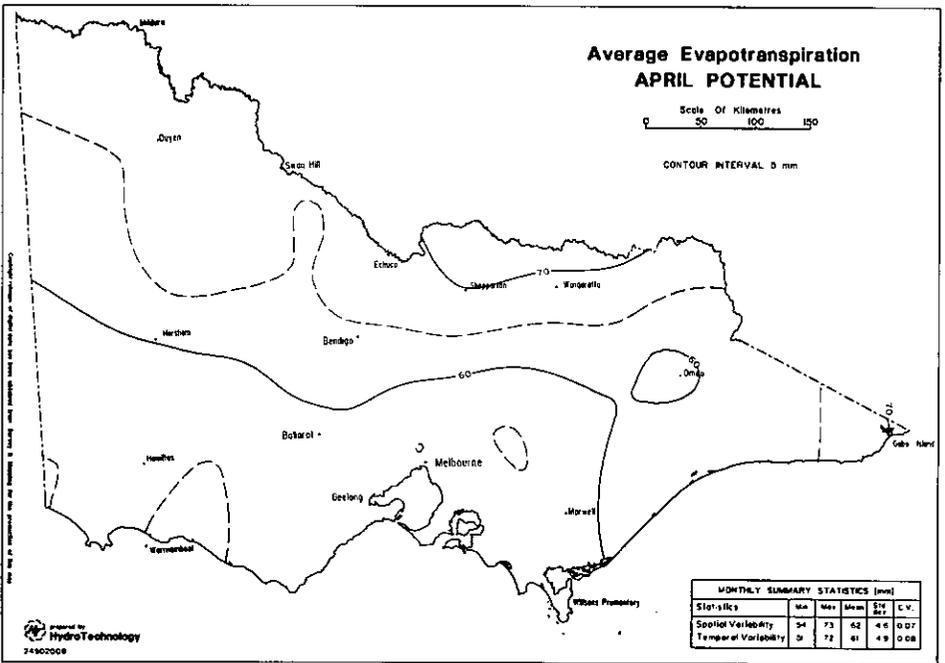
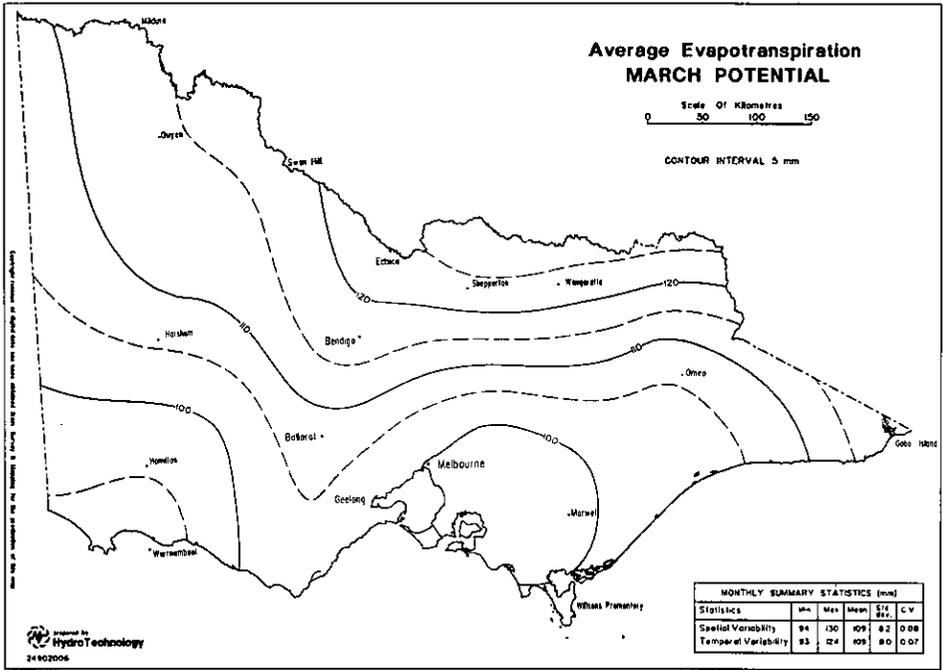
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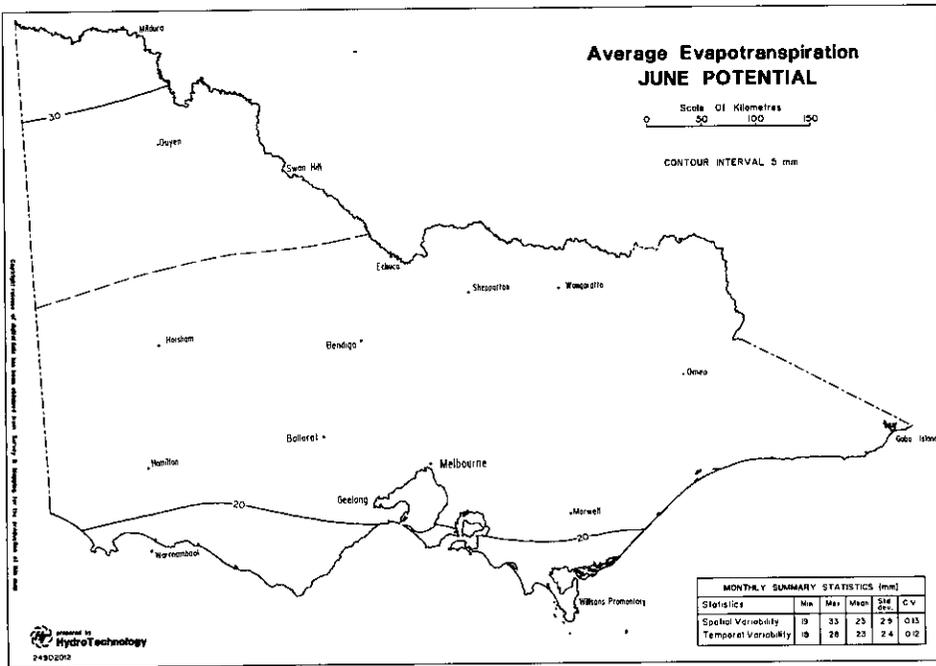
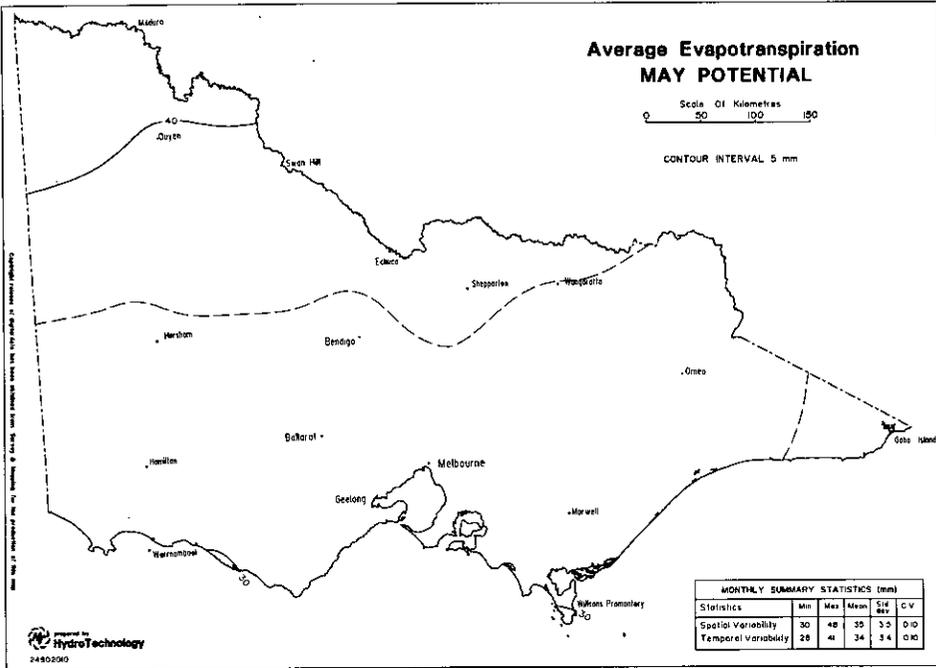
## Other Reading

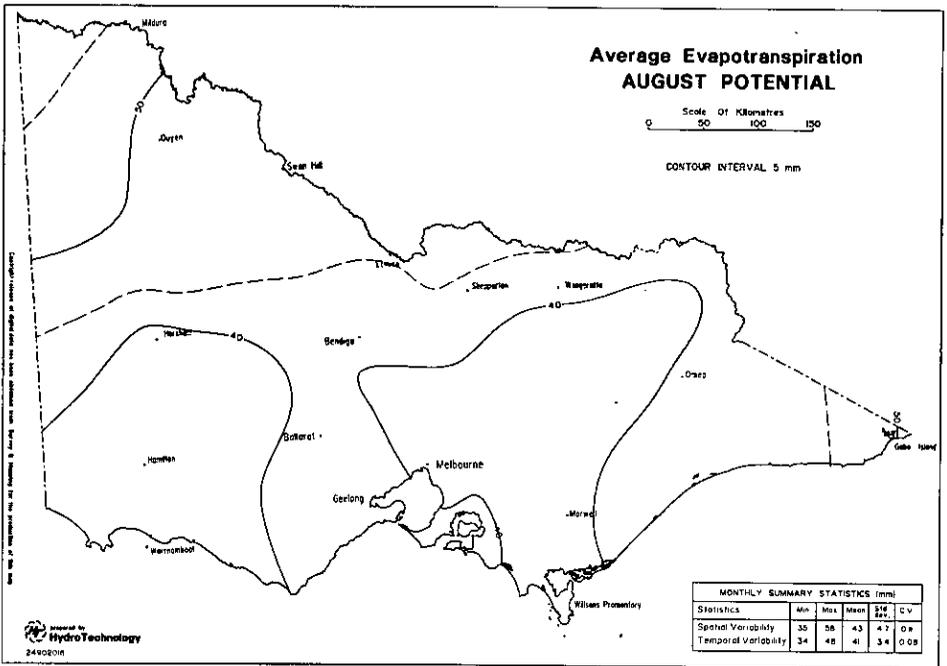
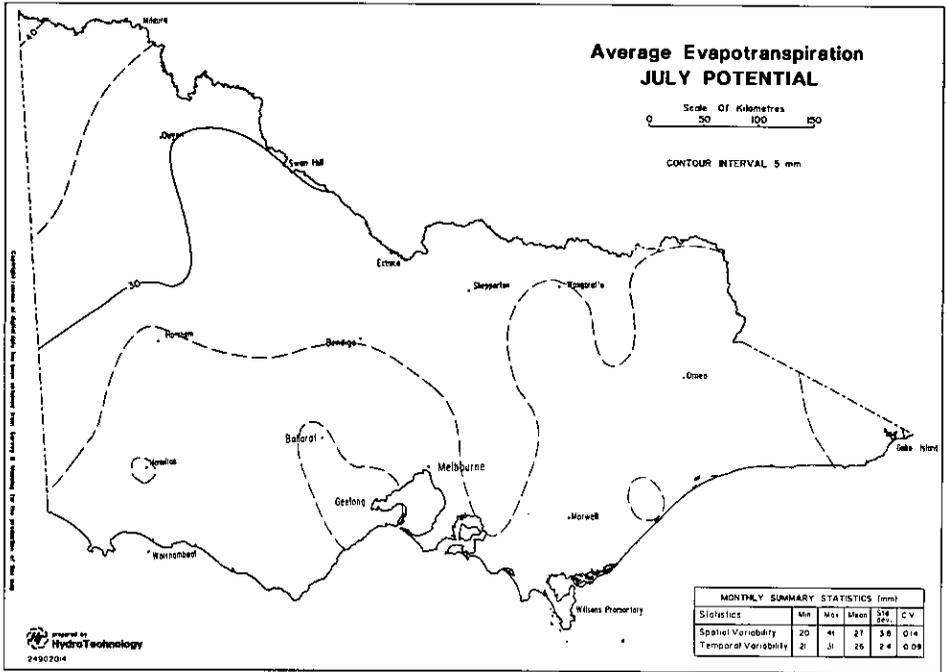
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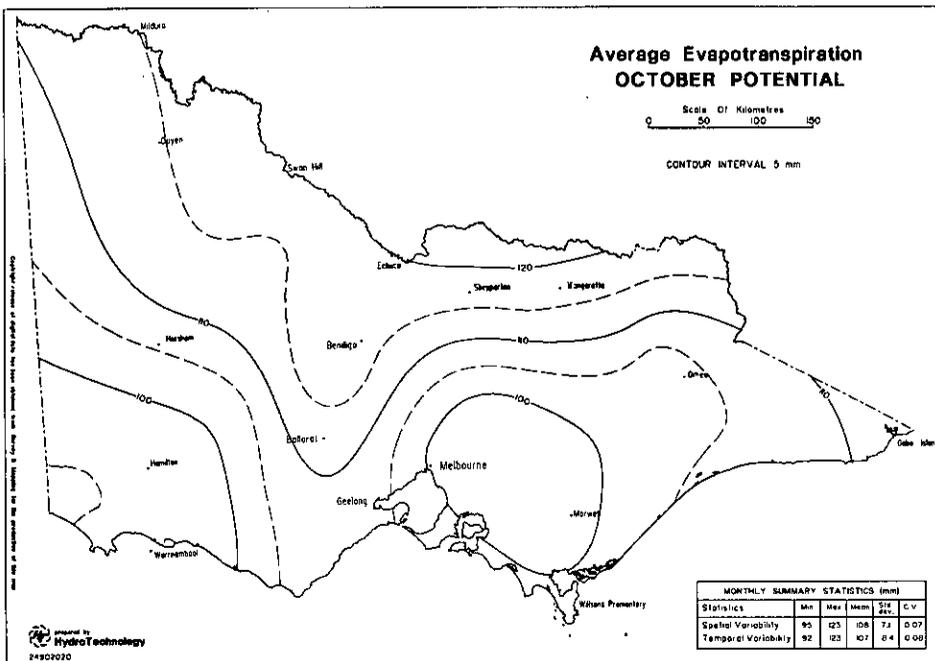
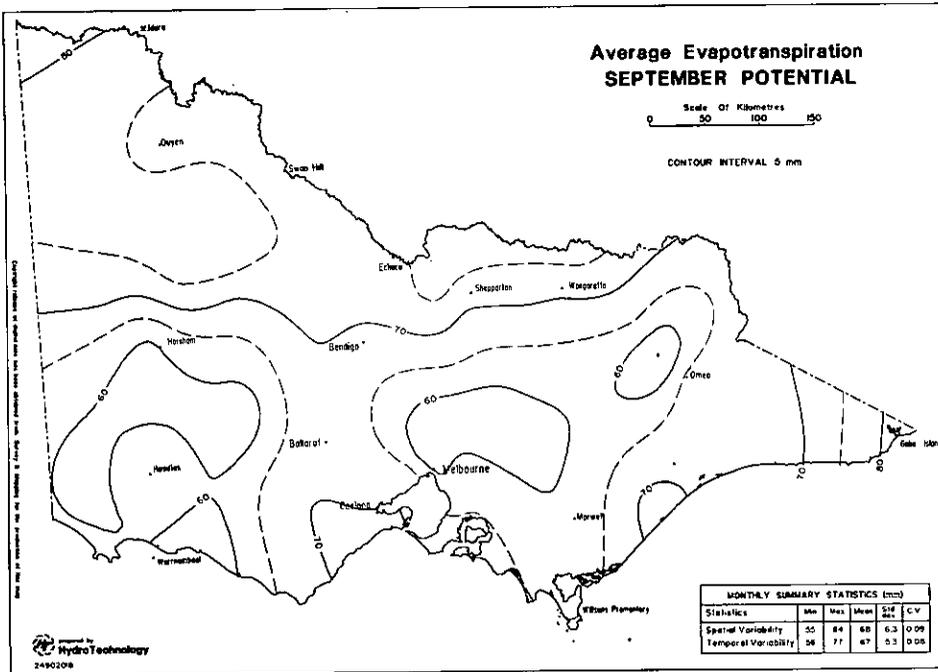


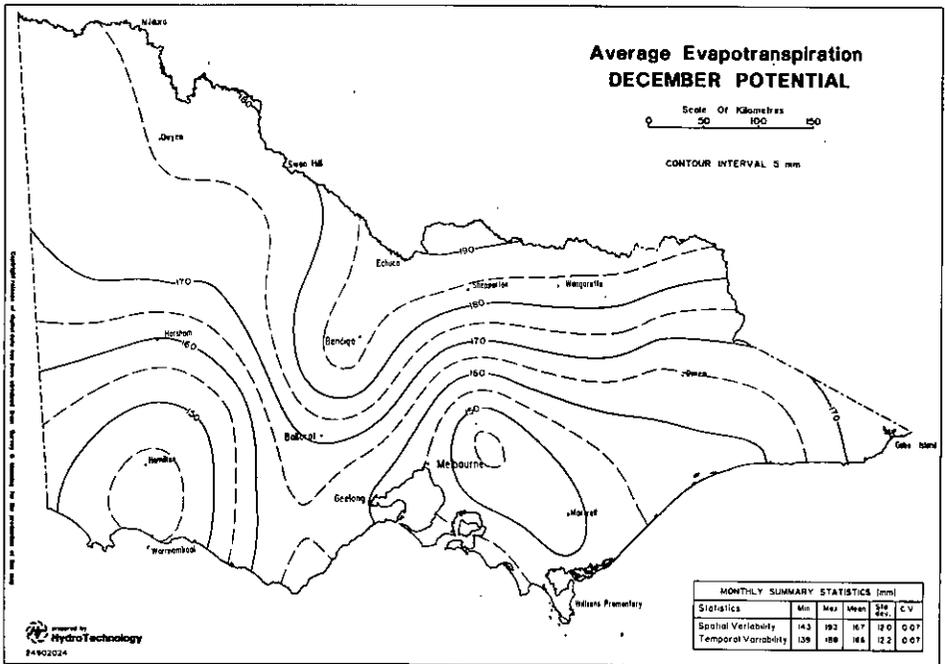
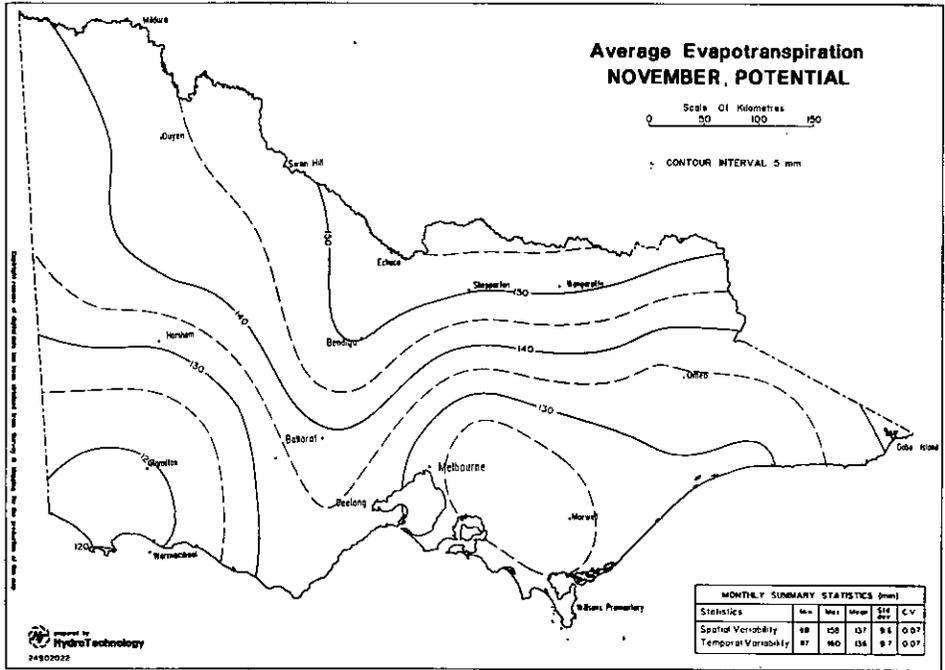












## 4.4 ESTIMATION OF PENMAN-MONTEITH $ET_0$ FROM CLASS A PAN DATA

Class A pan evaporation data are available for many meteorological stations around Australia. Unfortunately these data are not particularly accurate estimates of natural evaporation. Other more accurate methods for computing evapotranspiration are included in this handbook and should be used unless only pan data are available. Coefficients can be used to obtain estimates of actual evapotranspiration from class A pan data but these too are not reliable. In an effort to determine more realistic coefficients for Australia, Chiew et al. (1995) related Class A pan evaporation to Penman-Monteith  $ET_0$  as calculated in Topic 4.6 for 16 sites around Australia. This topic presents the results of that regression analysis and should enable better use of class A pan data.

### Calculation

$$ET_0 = G (\text{PAN}) \quad (4.4.1)$$

where

$ET_0$  Penman-Monteith Reference Crop Evapotranspiration

PAN Class A pan Evaporation

G The gradient of the  $ET_0$ -PAN regression line (Table 4.4.1)

Table 4.4.1 Values of G for each season for 16 climate stations

	Summer	Autumn	Winter	Spring
Alice Springs	0.62	0.61	0.61	0.64
Brisbane	0.78	0.75	0.66	0.74
Cairns	0.69	0.74	0.69	0.69
Canberra	0.67	0.65	0.61	0.68
Ceduna	0.68	0.68	0.69	0.70
Cobar	0.65	0.64	0.68	0.66
Giles	0.61	0.57	0.56	0.63
Halls Creek	0.67	0.63	0.60	0.62
Laverton	0.72	0.69	0.69	0.73
Mount Gambier	0.75	0.76	0.81	0.80
Mount Isa	0.67	0.65	0.65	0.65
Perth	0.73	0.67	0.64	0.77
Sydney	0.74	0.72	0.67	0.74
Tamworth	0.69	0.68	0.71	0.69
Tennant Creek	0.58	0.56	0.56	0.57
Woomera	0.63	0.63	0.64	0.66

### Error

Daily, 3-day, 5-day and 10-day Penman-Monteith evaporation values were compared with the Class A pan evaporation data for the sixteen stations listed above, with over

fourteen years of data for each station, and using a regression line forced through the origin. The mean (min; max) values of the coefficient of determination ( $R^2$ ) for the sixteen stations are summarised as:

Summer (Dec. - Feb.)

Daily	0.63 (0.36; 0.77)
3-day	0.75 (0.52; 0.85)
5-day	0.77 (0.57; 0.88)
10-day	0.77 (0.62; 0.90)

Autumn (Mar. - May)

Daily	0.68 (0.41; 0.82)
3-day	0.81 (0.57; 0.91)
5-day	0.84 (0.57; 0.93)
10-day	0.86 (0.56; 0.95)

Winter (Jun. - Aug.)

Daily	0.50 (0.11; 0.69)
3-day	0.68 (0.41; 0.85)
5-day	0.72 (0.45; 0.87)
10-day	0.72 (0.40; 0.89)

Spring (Sep. - Nov.)

Daily	0.67 (0.39; 0.78)
3-day	0.78 (0.50; 0.88)
5-day	0.80 (0.51; 0.90)
10-day	0.80 (0.47; 0.92)

The regression line gradient values, G, given in the table are for 3-day evaporation, as the mean values for periods greater than 3-day are not dissimilar.

It is recommended that the G values listed in the table are used only for calculating 3-day or longer evaporation, due to the relatively low values of the coefficient of determination for periods less than 3-day.

**Example**

Estimate monthly Penman-Monteith Reference Crop Evapotranspiration for Canberra (Station 070014) for October 1962, with measured pan evaporation of 152.9 mm.

From Table 4.4.1,  $G = 0.68$ .

Thus, (4.4.1), Penman-Monteith Reference Crop ET =  $0.68 * 152.9 = 104.0$  mm

## References

- Chiew, F.H.S., Kamaladasa, N.N., Malano, H.M. and T.A. McMahon (1995): Penman-Monteith, FAO-24 reference crop evapotranspiration and class-A pan data in Australia. *Agr. Water Management*, **28**, pp 9-21.
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## 4.5 REFERENCE CROP EVAPOTRANSPIRATION USING THE FAO - 24 RADIATION METHOD

This topic presents the FAO-24 Radiation method (FAO, 1984) for computing daily reference crop evapotranspiration ( $ET_0$ ). The international standard for estimating reference crop evapotranspiration is the Penman-Monteith method (see Topic 4.6) however the requirement for wind speed data often precludes its use. The FAO-24 Radiation method requires only average daily temperature and sunshine hours and estimates of long-term average relative humidity and daytime wind run. The method has been shown to give similar results to the full Penman-Monteith equation for Australian data (Chiew et al., 1995).

The required equation is:

$$ET_0 = c (W R'_s) \tag{4.5.1}$$

where

$ET_0$  reference crop evapotranspiration [ $mm\ d^{-1}$ ]

$R'_s$  shortwave radiation [ $mm\ d^{-1}$ ] computed from  $R_s$  as calculated in Topic 4.2

**Note** that  $R'_s = 0.408 R_s$  to convert the units from [ $MJ\ m^{-2}\ d^{-1}$ ] to [ $mm\ d^{-1}$ ]

$W$  is a factor which depends on average daily temperature and altitude and is given in Table 4.5.1.

Table 4.5.1 Values for factor  $W$  after FAO (1984)

Temp (°C)	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
W at altitude (m)																				
0	.43	.46	.49	.52	.55	.58	.61	.64	.66	.68	.71	.73	.75	.77	.78	.80	.82	.83	.84	.85
500	.45	.48	.51	.54	.57	.60	.62	.65	.67	.70	.72	.74	.76	.78	.79	.81	.82	.84	.85	.86
1000	.46	.49	.52	.55	.58	.61	.64	.66	.69	.71	.73	.75	.77	.79	.80	.82	.83	.85	.86	.87
2000	.49	.52	.55	.58	.61	.64	.66	.69	.71	.73	.75	.77	.79	.81	.82	.84	.85	.86	.87	.88
3000	.52	.55	.58	.61	.64	.66	.69	.71	.73	.75	.77	.79	.81	.82	.84	.85	.86	.88	.88	.89
4000	.55	.58	.61	.64	.66	.69	.71	.73	.76	.78	.79	.81	.83	.84	.85	.86	.88	.89	.90	.90

$c$  is a factor which can be determined using long term averages of wind speed and relative humidity and is implicitly derived using the graphs in Figure 4.5.1. Where actual data are not available, reasonable estimates of mean daytime windspeed ( $U_{daytime}$  in Figure 4.5.1) and relative humidity ( $RH$  in Figure 4.5.1) may be used without compromising the results.

Note (for those interested in evaporation theory): The term  $W$  in Equation 4.5.1 =  $\Delta/(\Delta+\gamma)$  as used in the Penman-Monteith, Priestly-Taylor and Moreton equations, while  $c$  is a correction coefficient factoring net radiation and advective effects.

The steps to computation  $ET_0$  are then:

1. Calculate  $R_s$  from Topic 4.2 and thus get  $R'_s = 0.408 R_s$
2. Calculate  $W$  using average daily temperature and Table 4.5.1.
3. Use Figure 4.5.1 and long term estimates of relative humidity and windspeed. Enter appropriate graph using computed  $W$   $R'_s$  and read off  $ET_0$ .

### Example

Calculate  $ET_0$  at Tatura on 1 October 1989 given the following:  
 (same conditions as the example in topic 4.6); latitude of 36.26 S ( -0.633 rad),  
 $T_{max}=16.3^\circ\text{C}$ ,  $T_{min} = 11.8^\circ\text{C}$  (i.e. mean temp =  $14.1^\circ\text{C}$ ), altitude 110 m,  $RH_{min} = 0.412$ ,  $RH_{max}= 0.764$ , wind run = 300 km/day =  $3.5 \text{ ms}^{-1}$  (note average daytime wind speed may be a little higher),  $R_s = 16.7 \text{ MJ m}^{-2} \text{ d}^{-1}$

1.  $R'_s = 0.408 R_s = 0.408 \times 16.7 = 6.81 \text{ mm d}^{-1}$
2. for altitude 110 m and  $T = 14.1^\circ\text{C}$ , from table 4.5.1,  $W = 0.61$  hence  $WR'_s = 4.15 \text{ mm d}^{-1}$
3. Enter upper left hand panel with wind speed level 2 and read off  $ET_0 = 3.7 \text{ mm d}^{-1}$

### *References*

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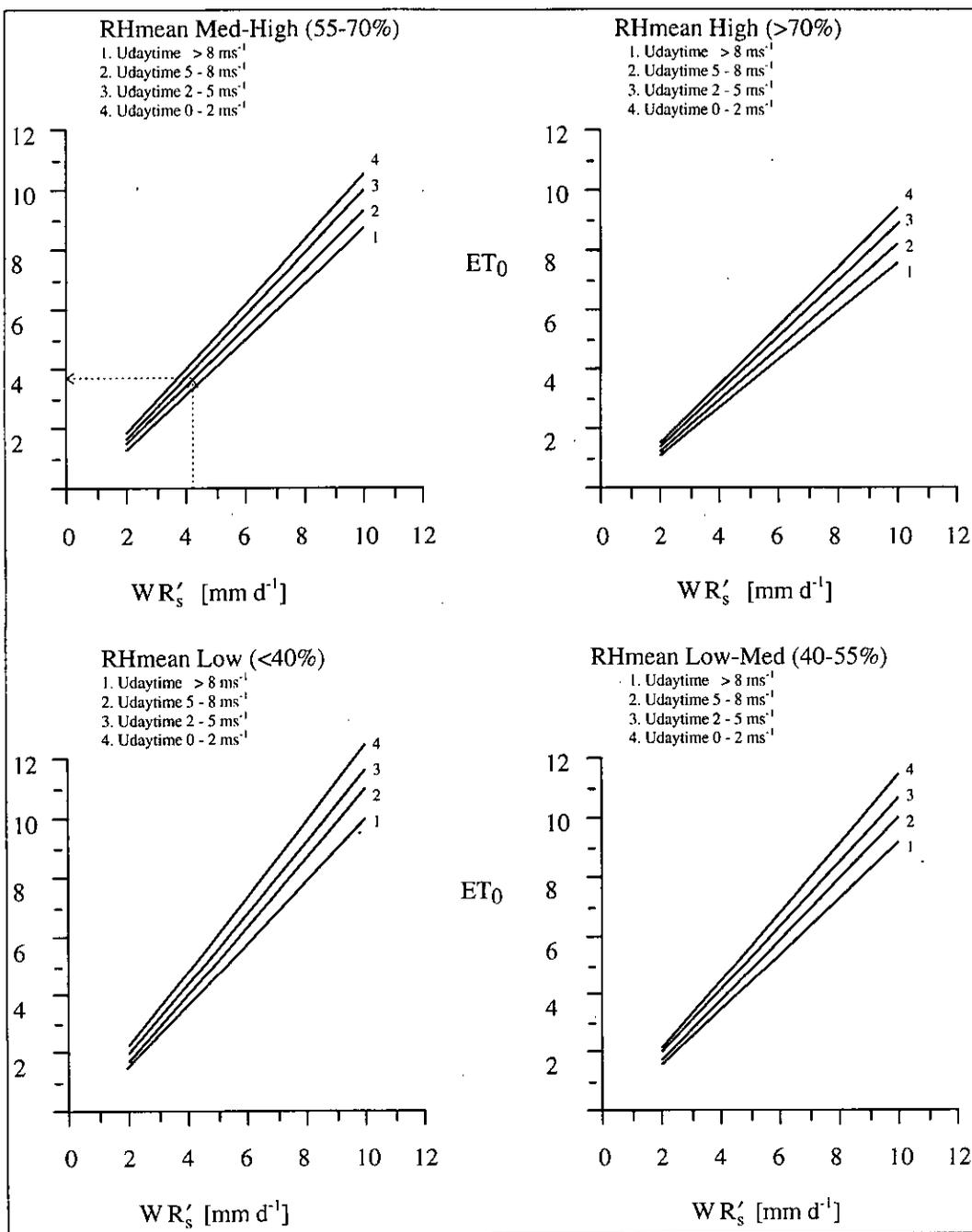


Figure 4.5.1  $WR'_s$  versus  $ET_0$  for various windspeed and humidity classes

## 4.6 EVAPOTRANSPIRATION USING THE PENMAN-MONTEITH METHOD

This topic presents the Penman-Monteith method for computing daily evapotranspiration (ET). It is considered as the international standard for estimating  $ET_0$  and can be used to estimate ET from other surfaces where appropriate information exists.

The Penman Monteith equation for reference crop evapotranspiration is:

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{T + 273} U_2 (e_a - e_d)}{\Delta + \gamma(1 + 0.34U_2)} \quad (4.6.1)$$

where

$ET_0$	reference crop evapotranspiration [ $\text{mm d}^{-1}$ ]
$R_n$	net radiation at crop surface [ $\text{MJ m}^{-2} \text{d}^{-1}$ ] - see Topic 4.2
$G$	soil heat flux [ $\text{MJ m}^{-2} \text{d}^{-1}$ ] - see (4.6.2)
$T$	average temperature [ $^{\circ}\text{C}$ ]
$U_2$	windspeed measured at 2m height [ $\text{m s}^{-1}$ ]
$(e_a - e_d)$	vapour pressure deficit [ $\text{kPa}$ ] - see (4.6.3)
$\Delta$	slope of vapour pressure curve [ $\text{kPa } ^{\circ}\text{C}^{-1}$ ] - see (4.6.4)
$\gamma$	psychrometric constant [ $\text{kPa } ^{\circ}\text{C}^{-1}$ ] $\approx 0.66$
900	conversion factor

Auxiliary equations:

$$G = c_s d_s \left( \frac{T_n - T_{n-1}}{\Delta t} \right) \quad (4.6.2)$$

where

$c_s$	volumetric heat capacity of soil [ $\text{MJ m}^{-3} ^{\circ}\text{C}^{-1}$ ] $\approx 2.1$ for average moist soil
$d_s$	estimated effective soil depth [m]
$T_n$	average temperature on day (or month) n
$T_{n-1}$	average temperature on preceding day (or month) n-1
$\Delta t$	length of period n [d]

Note:  $ET_0$  is sensitive only to  $G$  if there is a large temperature difference between two days. If information is not available,  $G$  may be set to 0.

$$e_a = 0.611 \exp\left(\frac{17.27T}{T + 237.3}\right) \quad (4.6.3)$$

$$e_d = RH e_a$$

where

$e_a$  saturation vapour pressure at temperature  $T$  [kPa]

$e_d$  actual vapour pressure [kPa]

RH relative humidity [fraction]

$T$  temperature [ $^{\circ}\text{C}$ ]

Ideally pairs of  $T$  and RH from throughout the 24 hour period are used to compute a range of  $(e_a - e_d)$  which are then averaged to give a daily value. If this is not possible, average values of  $e_d$  may be calculated as the mean of  $(RH e_a)$  at 0900 and 1500. Average  $e_a$  values may be calculated as the mean of  $e_a$  at  $T_{\max}$  and  $T_{\min}$  for the day.

$$\Delta = \frac{409.8e_a}{T + 237.3} \quad (4.6.4)$$

### **ET for other crops**

It is common for ET to be estimated for other crops by using 'crop factors' multiplied by  $ET_0$ . It is also possible to use Equation 4.6.1 to compute evapotranspiration for crops other than the reference crop by modification to the factors 0.34 and 900 in Equation 4.6.1 which are related to aerodynamic characteristics of the reference crop. Equation 4.6.1 can be expressed as:

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma \frac{185500 (e_a - e_d)}{(T + 273) r_a}}{\Delta + \gamma \left(1 + \frac{r_c}{r_a}\right)} \quad (4.6.5)$$

where

$r_c$  is the crop canopy resistance [ $\text{sm}^{-1}$ ] and  $\approx 200/\text{LAI}$ ; where LAI is the leaf area index

LAI  $\approx 24 \times$ (crop height in m) for clipped grass or LAI  $\approx 5.5 + 1.5 \ln$ (crop height in m) for alfalfa and other field crops

$r_a$  is the aerodynamic resistance [ $\text{sm}^{-1}$ ] and is given by:

$$r_a = \frac{\ln\left(\frac{z_m - d}{z_{om}}\right) \ln\left(\frac{z_h - d}{z_{oh}}\right)}{k^2 U_z} \quad (4.6.6)$$

where

$z_m$  height of windspeed measurements [m]

$z_h$	height of temperature and humidity measurements [m]
$k$	von Karman constant = 0.41
$U_z$	windspeed measurement at height $z_m$ [ $\text{ms}^{-1}$ ]
$d$	zero plane displacement $\approx 0.667x$ (crop height in m)
$z_{om}$	roughness height for momentum transfer $\approx 0.123x$ (crop height in m)
$z_{oh}$	roughness height for sensible heat transfer $\approx 0.1z_{om}$

### Example

Calculate  $ET_0$  at Tatura on 1 October 1989 given the following:  
 (same conditions as the example in Topics 4.4 and 4.5); latitude of 36.26 S (-0.633 rad),  
 $T_{1500} = 16.3^\circ\text{C}$ ,  $T_{0900} = 11.8^\circ\text{C}$  (i.e. mean temp =  $14.1^\circ\text{C}$ ),  $RH_{1500} = 0.412$ ,  $RH_{0900} = 0.764$ , wind run = 300 km/day =  $3.5 \text{ ms}^{-1}$ ;  $T_{\text{max}} = 18.1^\circ\text{C}$ ,  $T_{\text{min}} = 10.0^\circ\text{C}$

$R_n$	from Topic 4.2 = $9.0 \text{ MJ m}^{-2} \text{ d}^{-1}$
$G$	insufficient information is given so assume = 0
$T$	$14.1^\circ\text{C}$
$U_2$	$3.5 \text{ ms}^{-1}$
$\gamma$	0.66
$e_a$	from (4.6.3) at $T_{1500}$ , $e_a = 1.85 \text{ kPa}$ , at $T_{0900}$ $e_a = 1.38 \text{ kPa}$ , at $T_{\text{max}}$ , $e_a = 2.08 \text{ kPa}$ , $T_{\text{min}}$ , $e_a = 1.23 \text{ kPa}$ ; average $e_a$ for day = $(2.08 + 1.23)/2 = 1.66$
$e_d$	from (4.6.3), $RH e_a$ at 0900 = $0.764 \times 1.38 = 1.05$ and $RH e_a$ at 1500 = $0.412 \times 1.85 = 0.762$ ; average $e_d$ for the day = $(0.762 + 1.05)/2 = 0.91$
$e_a - e_d$	$1.66 - 0.91 = 0.75 \text{ kPa}$
$\Delta$	from (4.6.4), = $2.71 \text{ kPa}^\circ\text{C}^{-1}$

substituting into (4.6.1) gives  $ET_0 = 3.7 \text{ mm d}^{-1}$

### *References*

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 Development Division), Rome, 60pp.

## 4.7 EVAPORATION FROM OPEN WATER BODIES

This topic presents methods for computing evaporation from open water bodies such as lakes and ponds. This is of great practical importance to problems ranging from assessing evaporative losses from potable water storages to liquid waste control strategies that rely on evaporation such as tailings dams and evaporation ponds for saline water disposal. At first glance, this may seem to be the simplest of evaporation problems without the complexities of soil and vegetative surfaces. In practice, this is not the case and no single ideal method has been developed, at least not for use with commonly available data.

There are several key problems that arise. Firstly, the heat storage of a water body has a large effect on the surface energy exchange yet it is difficult to determine short term changes because the depth of mixing of the water varies in both time and space, as does the extent to which there is heat transfer across the lake bed boundary. Secondly, the clarity of the water body will affect the amount of heating and particularly the depth of heating. Turbid water often absorbs less heat but the very shallow depth of heating means actual loss rates are enhanced. Thirdly, the salinity (or concentration of other solutes) affects the evaporative process.

This topic presents methods that have been used in practice in both Australia and overseas. **It should be stressed that unlike reference crop evapotranspiration ( $ET_0$ ) where internationally recognised standard procedures exist, accurate assessment of open water evaporation is difficult and in important applications, the assistance of an expert in the field will be warranted.**

### *Class A Pan Coefficients*

The simplest approach to estimating open water evaporation is by using data from Class A Pans and a coefficient in an analogous way to Topic 4.4. Intuitively, one might expect this method to be more accurate when used for open water evaporation than when used to estimate  $ET_0$ , but this is not necessarily the case. Problems with pan exposure and heat storage effects render the approach a "last resort".

Hoy (1977) presented annual pan coefficients for a number of lakes around Australia, computed by comparing Class A Pan data to lake evaporation estimates made by water balance and energy budget methods (Table 4.7.1). The values show a wider range than the review of local and overseas studies by AWRC (1970) in which a pan coefficient of  $0.7 \pm 0.1$  was recommended. This illustrates the errors likely with an annual method.

If the approach is to be used, the pan should have a similar exposure to the lake and have an airflow that will be unaffected by the lake. It should also be used only for annual estimates where the assumption of zero net heat storage is more likely to be valid. In some cases, monthly pan coefficients have been derived for particular water bodies and these show substantial seasonal variation (e.g. Hoy and Stephens, 1977, 1979; Fleming et al., 1989)

Table 4.7.1 Computed Class A Pan Coefficients (after Hoy, 1977)

Water Body	Latitude (S)	Longitude (E)	Annual Pan Coefficient
Lake Menindee	32° 20'	142° 20'	0.71
Lake Pamamaroo	32° 28'	142° 28'	0.66
Lake Cawndilla	32° 28'	142° 14'	0.71
Stephens Ck. Res.	31° 50'	141° 30'	0.69
Lake Albacutya	35° 45'	141° 58'	0.79
Lake Hindmarsh	36° 5'	141° 55'	0.74
Lake Eucumbene	36° 5'	148° 40'	0.81
Cataract Res.	34° 20'	150° 50'	0.92
Manton Res.	12° 50'	131° 5'	0.87
Mundaring Res.	31° 55'	160° 10'	0.93
Blue Lagoon	38° 11'	146° 22'	0.88
Lake Wyangan Sth.	34° 17'	146° 2'	0.78
Rifle Ck. Res.	20° 57'	139° 35'	0.70

### Combination Equation

The combination equation based on Penman-type formulations can be used to estimate open water evaporation (e.g. Fleming et al., 1989). It requires the following data: net radiation, wind run at height of 2 m and pairs of temperature and humidity records. The equation is given below for daily computation but other time periods can be used by averaging the components appropriately. In general, aggregation to weekly or monthly values is desirable. It is interesting to note that the earliest combination style equation for open water evaporation was developed in 1944 (before Penman's work) by Ferguson of ICI for use on evaporation ponds in Adelaide.

The basic combination equation ignoring heat storage effects can be written as:

$$E_0 = 0.408 \left[ \frac{\Delta}{\Delta + \gamma} R_n + \frac{\gamma}{\Delta + \gamma} E_a \right] \quad (4.7.1)$$

where

- $E_0$  open water evaporation [ $\text{mm d}^{-1}$ ]
- $R_n$  net radiation at water surface [ $\text{MJ m}^{-2} \text{d}^{-1}$ ] - see Topic 4.2
- $E_a$  is a function of windspeed, saturation vapour pressure and average vapour pressure (see Equation 4.7.2) [ $\text{mm}$ ]
- $\Delta$  slope of vapour pressure curve [ $\text{kPa } ^\circ\text{C}^{-1}$ ] - see Topic 4.6
- $\gamma$  psychrometric constant [ $\text{kPa } ^\circ\text{C}^{-1}$ ]  $\approx 0.66$

$$E_a = f(u) (e_a - e_d) \quad (4.7.2)$$

where

$(e_a - e_d)$  vapour pressure deficit [kPa] - see Topic 4.6

$f(u)$  windspeed dependent transfer function given by Penman (1956) as  
 $= 2.6 (0.5 + u/161)$  where  $u$  is the windspeed at a height of 2 m [ $\text{km d}^{-1}$ ]

If wind data are not available from 2m, the following equation may be used to convert values from other heights ( $z_m$ ):

$$U_2 = U_{z_m} \frac{\ln \left[ \frac{2}{z_{om}} \right]}{\ln \left[ \frac{z_m}{z_{om}} \right]}$$

where

$z_{om} \approx 0.0023$  m for open water (Brutseart, 1982)

NOTES:

- the windspeed function has been the subject of many studies and alternative forms are available. The one above has been found to give satisfactory results in Australia and Botswana (Fleming et al., 1989)
- ignoring heat storage effects will render the method inaccurate for deep, clear water bodies and where high turbidity creates a shallow, hot layer of water.

### *Morton's Method*

This method does not require the windspeed data of the Penman formulations (it uses average dew point temperature, average max. and min. air temperature and the ratio of observed to maximum possible sunshine duration, all for periods of 5 days to one month) and is based on somewhat modified assumptions to Penman's formulation (see also Topics 4.1 and 4.3). It is assumed that the air mass passing over a surface is influenced by that surface and so effects the rate of evaporation. Morton argues that measurements made for use in the combination formula are measured in a "dry" environment yet are being applied to a wet (lake) environment. There is much argument in the literature regarding Morton's method (Morton, 1983; Morton, 1986) but it is gradually gaining greater acceptance, particularly for estimation of evaporation over large areas.

The method is available as a computer model, the "Complementary Relationship Lake Evaporation" model, CRLE (Morton et. al., 1986). Full descriptions of the method are available in the references.

### *Heat Storage effects*

If heat storage effects of reservoirs need to be represented, Morton (1986) presents a simple method for incorporation into his model. While the formulation is simplistic, it effectively represents the lag in evaporation caused by seasonal heat storage effects. The method uses a routing approach through a hypothetical heat storage reservoir with delay times and storage constants made a function of lake depth and salinity. The quantity routed is the solar and waterborne heat input ( $G_W^0$ )

$$G_W^0 = (1 - \alpha)G + h \quad (4.7.3)$$

where

- $\alpha$  surface albedo = 0.05 for a water surface
- $G$  incident global radiation
- $h$  waterborne energy input which is generally negligible unless the temperature of inflows significantly differ from those of outflows and the inflow volumes are large.

Conceptually, this kind of approach could be utilised in other models of open water evaporation.

### *Effects of Salinity*

Morton also has a method for adjusting evaporation rate on the basis of water salinity (Morton et. al., 1985):

$$E = \frac{EW(\text{Fresh})}{1 + \left(\frac{\text{salt}}{10^6}\right)} \quad (4.7.4)$$

where

$E$  lake evaporation [mm]

salt salinity [ppm]

$EW(\text{fresh})$  fresh water lake evaporation [mm]

Note that salinity is often measured in E.C. (electrical conductivity) units e.g.,  $\mu\text{S}/\text{cm}$ . The conversion from E.C. to a concentration of dissolved solids depends on the ionic composition of the water. For water where unusual ionic compositions are not expected, it is common to assume that the concentration of total dissolved solids [ppm or mg/L] =  $0.6 \times \text{E.C.}$  [ $\mu\text{S}/\text{cm}$ ] (Hart and McKelvie, 1986).

Again it is possible to use the Equation 4.7.4 with other models of open water evaporation by substituting the initial estimate of evaporation for EW(fresh) to give a value adjusted for salinity (E).

## References

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## 5.1 GENERATION OF ANNUAL AND MONTHLY TIME SERIES

### *First Order Markov Models for generating Annual Series*

The field of stochastic data generation is extensive and includes problems ranging from single site streamflow models with stationary statistical properties to the multi-site generation of rainfalls and correlated evaporation incorporating non-stationarity and the possibility of catastrophic events. The primary aim with all methods is to compute a data series that has similar statistical properties to a measured set of data. Such a data series is called "synthetic" data. Because it can extend over a much greater period than measured data, it enables a wider range of conditions to be considered. It is in this sense that synthetic data make fullest use of the information in the measured data.

In this topic, a simple model for the generation of monthly and annual time series (eg. rainfall, streamflow, evaporation etc.) is presented. It is assumed that statistical properties are stationary (i.e. do not change over time) or can be made so by simple transformation. It must be recognised that "synthetic" data, no matter how it is computed, cannot improve poor records but merely improve the designs made with whatever data are available. It is worth considering Table 5.1.1, in which minimum record lengths of annual flows needed to estimate the mean annual flow (MAF) to a given standard error for different coefficient of variations of annual flows.

Table 5.1.1 Years of record needed to obtain a given standard error in mean annual runoff for different  $C_v$

Std. error of MAF	Coefficient of variation ( $C_v$ ) of mean annual runoff			
	0.25	0.5	1	1.5
0.05 (5%)	25	100	400	900
0.1 (10%)	7	25	100	225
0.25 (25%)	1	4	16	36
0.5 (50%)	-	1	4	9

Time series can be considered to comprise of five components: trend, periodic or seasonal, short term memory or persistence, random and long memory. In the method presented here, trend is not considered although can be incorporated (with various levels of error) through the use of polynomials or moving averages. Short term memory or persistence represents the extent to which a data value depends on its previous value (lag one) or two values previous (lag two) and so on. In the model presented here, only lag one persistence is considered.

For any analysis, historic data must be used to compute basic statistics such as mean, standard deviation and lag one serial correlation coefficient (the correlation coefficient for a value of flow versus the previous value, for all flow records). The serial correlation coefficient ( $r_k$ ) should be checked to ensure it is significantly different from zero (see Topic 5.5).

### *Markov Annual Flow Model*

$$x_{i+1} = \bar{x} + r_1(x_i - \bar{x}) + t_i s(1 - r_1)^{0.5} \quad (5.1.1)$$

where

- $x_{i+1}, x_i$     annual values for years (i+1) and i
- $s$              standard deviation of annual data
- $r_1$             annual lag one serial correlation coefficient
- $t_i$             normal random variate with a mean of zero and a variance of unity

In this formulation, it is assumed that the data are normally distributed. This is often not the case and non-normality can be incorporated by replacing  $t_i$  with  $t_\gamma$  using the method described later.

The normal variate  $t_i$  can be generated using a pseudo random number generator on a computer. Often these are uniformly distributed with a mean of 0.5 and a variance of 1/12. If 12 of these numbers are added together and 6 is subtracted, the resulting number can be considered as a normal random variate of variance unity. Alternatively, more sophisticated number generators could be used (e.g. "Numerical Recipes" by Press et al., 1992).

To initiate the model, begin with  $x_1$  equal to the mean value but discard the first 10 or so flows since they will be dependent on the initial condition. It is possible for negative values to be generated. If this occurs, discard the number and resample, but keep a track of how many times a negative number is produced. This procedure is acceptable so long as the number of negative values does not exceed 5%.

### *Modification for Non-Normal Data*

Both the Box-Cox (B-C) and Wilson-Hilferty (W-H) transformations can be used to convert non-normal data to a form suitable for analysis using the approach outlined above. The B-C method (McMahon and Mein, 1986) is applicable over a wide range of skewness but is more complex than W-H.

If  $C_S < (4 - 3.3 r_1)$ , (where  $C_S$  is the coefficient of skewness for annual flows and  $r_1$  is the annual lag one serial correlation coefficient), it is possible to use W-H. If the data is more highly skewed, B-C must be used and readers should consult the references.

Using the W-H transformation, non-normal data are included in the annual model by replacing  $t_j$  with  $t_\gamma$  to produce a "like gamma" variate:

$$t_\gamma = \frac{2}{\gamma_t} \left[ 1 + \frac{\gamma_t t_j}{6} - \frac{\gamma_t^2}{36} \right]^3 - \frac{2}{\gamma_t} \tag{5.1.2}$$

where

$\gamma_t$  coefficient of skewness of the like Gamma variate and calculated as in (5.1.3)

$t_\gamma$  "like Gamma" variate

$$\gamma_t = \frac{\gamma - r_1^3 \gamma}{(1 - r_1^2)^{1.5}} \tag{5.1.3}$$

where

$\gamma$  coefficient of skewness of annual data

$r_1$  annual lag 1 correlation coefficient

Note: Whenever data are generated, it is very important to perform diagnostic checks to show that the model assumptions are reasonable and that there are no computational errors. This is easiest done by computing the statistics of the generated data series and then comparing these with the statistics of the original data to ensure that they are consistent.

### Monthly Generation using the Method of Fragments

Observed monthly data are standardised by dividing each monthly value by the corresponding annual sum. The resulting set of standardised monthly data for each year are referred to as fragments ( $f$ ). Thus for a given year:

$$\sum_{j=1}^{12} f_j = 1 \tag{5.1.4}$$

Table 5.1.2 shows five years of monthly data with the fragments calculated for each month.

Annual data from the historic record ( $N$  years) are then ranked according to increasing magnitude and  $N$  classes are formed. The lower limit of Class 1 is zero while there is no upper limit of Class  $N$ . Intermediate Class limits are the average of two successive

values in the ranked series. The number of annual data would generally be large but for illustrative purposes, consider the data in Table 5.1.2 which are ranked as follows:

- 55050 (1991)
- 71340 (1989)
- 91100 (1990)
- 156070 (1988)
- 161500 (1992)

Thus class 1 is 0 to  $(55050+71340)/2 = 63195$ , class 2 is 63195 to  $(71340+91100)/2 = 81220$  and so on to class 5 which is greater than  $(156070+161500)/2 = 158785$ .

The fragments obtained from the monthly values contributing to the smallest annual sum are assigned to class 1 and those from the monthly values contributing to the second smallest annual sum are assigned to class 2 and so on. Annual data are then generated by a method such as that described above and each value is checked to determine its class. The appropriate set of fragments can then be applied to the relevant annual flow to give monthly values. For example, if the generated value was 80050, it would be in class 2 and the fragments from 1989 would be multiplied by 80050 to give monthly values.

Table 5.1.2 Flows (ML) and calculated fragments for five years of monthly data

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	TOTAL
1988	6400 0.04	2080 0.01	1540 0.01	1850 0.01	6640 0.04	18280 0.12	36720 0.24	25880 0.17	29720 0.19	10920 0.07	8040 0.05	8000 0.05	156070
1989	2880 0.04	2520 0.04	2320 0.03	4560 0.06	6920 0.10	5800 0.08	6740 0.09	7240 0.10	9400 0.13	13680 0.20	7040 0.10	2240 0.03	71340
1990	6080 0.07	1240 0.01	500 0.01	1000 0.01	3840 0.04	6200 0.07	13800 0.15	16480 0.18	19640 0.21	11200 0.12	6880 0.08	4240 0.05	91100
1991	1800 0.03	880 0.02	5520 0.10	5520 0.10	6080 0.11	6120 0.11	4570 0.08	7360 0.13	8360 0.16	6240 0.11	1760 0.03	840 0.02	55050
1992	1200 0.01	900 0.01	4160 0.03	5360 0.03	11280 0.07	29160 0.17	24200 0.15	14360 0.09	40440 0.24	20480 0.13	7360 0.05	2600 0.02	161500

### Advanced Data Generation

The approaches presented in this topic are at the simple end of a broad spectrum of stochastic generation techniques. More advanced methods which deal with multi site generation and correlated data are discussed in McMahon and Mein (1986). Robust

estimation methods for multi-site models in the presence of incomplete and missing data are presented in Kuczera (1987). There are also other methods for establishing the statistics of populations from sub-samples such as bootstrapping (Efron and Tibshirani, 1993) which should be considered. When data records are short, the parameter uncertainty may become large (Table 5.1.1). The reader is referred to Taylor and Stedinger (1982) for a treatment of this problem.

## References

- Efron, B. and R.J. Tibshirani, 1993. *An Introduction to the Bootstrap*. Chapman and Hall, New York, 436pp.
- Kuczera, G. (1987): On maximum likelihood estimators for the multisite lag-one streamflow model: Complete and incomplete data cases. *Water Resour. Res.*, **23(4)**, pp. 641-645.
- McMahon, T.A. and R.G. Mein (1986): *River and Reservoir Yield*. Water Resources Publications, Littleton, Colorado, 368pp.
- Press, W.H., Teukolsky, S.A., Vetterling, W.T. and B.P. Flannery (1992): *Numerical Recipes in C - The art of scientific computing*. Cambridge University Press, 994pp.
- Taylor, M.R. and J.R. Stedinger (1982): Synthetic streamflow generation, 2: Effect of parameter uncertainty. *Water Resour. Res.*, **18(4)**, pp. 919-924.

## 5.2 AN APPROACH TO SIZING STORAGE PONDS BASED ON GENERATED DATA

Storage ponds, particularly those built for waste disposal, must often be designed with a particular risk of "failure" i.e., with some specified probability of discharging during a specified period. The problem is therefore one of balancing the inputs (rainfall, groundwater inflows and waste discharge) against the outputs (evaporation and groundwater leakage). It might also be the case that the waste contains solids and so the available volume is reducing over time. A method for solving this type of problem, using generated data sequences, is presented here.

### *Method*

Sequences of monthly evaporation and rainfall data (say 100 sequences) are generated as described in Topic 5.1. Each of these is  $n$  years long where  $n$  is the design life of the storage. The requirement then is to size the storage such that there is a risk  $r$  of one or more overflows from the pond during the  $n$  year period.

The next step is a month by month simulation of the pond storage, taking into account the inputs (rainfall, waste discharge) and the outputs (evaporation, possibly seepage, overflows). At least two methods of analysis are then possible:

- (i) Determine for each sequence, the storage size for which there would be **one** overflow during the  $n$  year period. This can be done automatically using a procedure analogous to the sequent peak method (McMahon and Mein, 1986, p.161). The resulting 100 storage sizes are then ranked and the required size is that for which there is a proportion  $r$  larger.
- (ii) Route each sequence in turn through a pond of particular size (starting at empty each time) and determine the proportion of sequences that cause an overflow. This must be done iteratively (i.e. with different sized ponds) until a capacity for which overflows occur in a proportion  $r$  of the 100 sequences is found. The iterative step may also be done in terms of having a fixed pond size and trying different discharge volumes to give a proportion  $r$  of failures.

Method (ii) is better suited to some types of problems e.g., if it is required to determine waste discharge volumes that are possible for a fixed capacity holding pond.

### *References*

McMahon, T.A. and R.G. Mein (1986): *River and Reservoir Yield*. Water Resources Publications, Littleton, Colorado, 368pp.

## 5.3 ESTIMATION OF RESERVOIR YIELD

### *Gould Gamma Method*

The Gould Gamma technique (McMahon and Mein, 1986) can be used for the calculation of reservoir draft, or yield, as a function of mean annual flow, coefficient of variation of annual flows, storage volume and probability of being empty in any year. It is applicable for sites involving carry-over storage (i.e., reservoirs which spill, on average, much less than annually).

#### Calculation

$$D = 1 - \frac{z_p^2 C_v^2}{4(\tau + dC_v^2)} \quad (5.3.1)$$

where

- D draft or yield expressed as a ratio of mean annual flow  
 $z_p$  standardised normal variate for a design probability of failure  
 $C_v$  coefficient of variation of annual flows (= standard deviation/mean ann. flow)  
 $\tau$  = storage / mean annual flow  
d correction factor  
p annual probability of failure (= 1 - reliability)

Table 5.3.1 Correction factors and standardised normal variates for  $p = 1, 2, 5\%$

$p(\%)$	$z_p$	d
1	2.33	1.5
2	2.05	1.1
3	1.88	0.9
4	1.75	0.8
5	1.64	0.6

#### **Assumptions**

- uniform draft rate
- annual flows are independent
- n-year flow sums are Gamma distributed
- no evaporation (see adjustment procedure overleaf)

**Check Carry Over Criterion (to ensure method is valid)**

The carry-over criterion is determined by the value of  $m$  (Vogel and Stedinger, 1987), calculated by:

$$m = \frac{1-D}{C_v} \tag{5.3.2}$$

If  $m \geq 1$       only within-year storage (Gould Gamma method may not be used).  
 $m < 1$         carry-over storage (Gould Gamma method may be used).

**Required Storage**

$$\tau = \left\{ \frac{z_p^2}{4(1-D)} - d \right\} C_v^2 \tag{5.3.3}$$

For a mean annual flow of  $\bar{X}$ , the storage,  $C$ , required to achieve  $\tau$  is

$$C = \tau \bar{X} \tag{5.3.4}$$

**Adjustment for Evaporation**

$$\Delta C_E = 0.7 A_F \Delta E_p C_p \tag{5.3.5}$$

where

- $\Delta C_E$       additional storage to cover evaporation losses [ $m^3$ ]
- $A_F$         surface area of reservoir at full supply [ $m^2$ ]
- $\Delta E_p$       net evaporation loss [m/year]
- $C_p$         critical drawdown period [years]

**Critical Period**

The period during which a reservoir goes from a full condition to an empty condition without spilling in the intervening period is given by

$$C_p = \left\{ \frac{z_p^2}{4(1-D)^2} \right\} C_v^2 \tag{5.3.6}$$

**Example**

A 600 GL storage is proposed for the Clarence River at Tabulam (Station 204900). At this site the mean annual flow is 912 GL and the standard deviation of annual flows is 802 GL. What annual yield could be sustained, with 95% reliability, for such a storage?

A storage of 600 GL at this site would have a  $\tau$  of  $600/912 = 0.66$ .

The river  $C_v = (802/912) = 0.88$

Thus, a draft with a 95% reliability ( $z_p = 1.64$ ,  $d = 0.6$ ) would be (5.3.1) :

$$\begin{aligned} D &= 1 - [(1.64^2 \cdot 0.88^2) / \{4(0.66 + 0.6 \cdot 0.88^2)\}] \\ &= 0.54 \end{aligned}$$

This is equivalent to an annual yield of  $0.54 \times 912 = 490$  GL.

Check for applicability of the Gould Gamma method by using Equation 5.3.2

$$m = (1 - 0.54) / 0.88 = 0.52 < 1$$

Thus the Gould Gamma calculations are applicable.

*References*

- McMahon, T.A. and R.G. Mein (1986): *River and Reservoir Yield*. Water Resources Publications, Littleton, Colorado, 368pp.
- Vogel, R.M. and J.R. Stedinger (1987): Generalized storage-reliability-yield relationships. *Jour. of Hydrology*, **89**, pp. 303-327.

## 5.4 PROBABILITY OF n-YEAR FLOW SEQUENCES

### *Distribution of n-Year Flows*

For an N year record of annual flows, it is sometimes desired to determine the probability of occurrence of an n-year flow sequence or flow sum,  $Y_n$ . This is often necessary when assessing the probability of a string of low flow years, and hence critical conditions for water supply.

Since the flow sequence is being considered as it occurs within the flow record, the sum comes from what is known as an overlapping series. For easier statistical analysis, the sum requires factoring by a ratio,  $R_a$ , to produce an equivalent non-overlapping sum. The procedure presented in this topic (McMahon and Mein, 1986) is based upon the determination of an appropriate distribution for the flow series and the calculation and adjustment of the series parameters.

### Determination of Distribution

The coefficient of skewness,  $C_s$ , (5.4.1) of the N year series of annual flows ( $x_i$ ) is calculated to determine if the series has a Normal ( $C_s \approx 0$ ) or Gamma ( $C_s \approx 2C_v$ ) distribution.

$$C_s = a / s^3 \quad (5.4.1)$$

where 
$$a = [N/(N-1)(N-2)] \sum_{i=1}^N (x_i - \bar{x})^3 \quad (5.4.2)$$

s standard deviation, given by 
$$\left[ \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2 \right]^{0.5} \quad (5.4.3)$$

$\bar{x}$  mean of annual flows.

with 
$$C_v = \text{coefficient of variation, given by } s/\bar{x} \quad (5.4.4)$$

If the value of  $C_s$  indicates that the data do not follow a Normal or Gamma distribution, then the transition probability matrix method of McMahon and Mein (1986) should be used in preference to the procedure described here.

### Calculation of Statistical Variables

Given normally or Gamma distributed flows, calculate as appropriate mean,  $\mu$ , variance,  $\sigma^2$ , shape parameter,  $\alpha$ , and scale parameter,  $\beta$ , using Equations 5.4.5 - 5.4.8. For normally distributed flow data, the flows must be standardised to a mean of 10 and a variance of 1.

**Normal** 
$$\mu = \bar{x} \quad (5.4.5)$$

$$\sigma^2 = s^2 \quad (5.4.6)$$

$$\text{Gamma} \quad \alpha = \frac{1 + \left(1 + \frac{4A}{3}\right)^{0.5}}{4A}, \text{ where } A = \log_e \bar{x} - \overline{\log_e x} \quad (5.4.7)$$

$$\beta = \bar{x} / \alpha \quad (5.4.8)$$

For an n year sequence, assuming independent annual flow values (see Topic 5.5 for test of significant auto-correlation), calculate the values of  $\mu_n$  and  $\sigma_n$ , or  $\alpha_n$  and  $\beta_n$ , using Equations 5.4.9 - 5.4.12.

$$\text{Normal} \quad \mu_n = n\mu \quad (5.4.9)$$

$$\sigma_n^2 = n\sigma^2 \quad (5.4.10)$$

$$\text{Gamma} \quad \alpha_n = n\alpha \quad (5.4.11)$$

$$\beta_n = \beta \quad (5.4.12)$$

If the flows are not independent, then use:

$$\text{Normal} \quad \mu_n = n\mu \quad (5.4.13)$$

$$\sigma_n^2 = R_n n \sigma^2 \quad (5.4.14)$$

$$\text{Gamma} \quad \alpha_n = n\alpha / R_n \quad (5.4.15)$$

$$\beta_n = R_n \beta \quad (5.4.16)$$

$$\text{where} \quad R_n = \frac{1+r}{1-r} - \frac{2r(1-r^n)}{n(1-r)^2};$$

where  $r$  is the lag one auto-correlation coefficient (see Topic 5.5).

### Modification for Overlapping Events

Having determined the values of  $\mu_n$  and  $\sigma_n$ , or  $\alpha_n$  and  $\beta_n$ , use Figures 5.4.1-5.4.3 to determine an appropriate value for the ratio,  $R_a$ , of non-overlapping sums to overlapping sums.

Calculate the non-overlapping sum,  $Z_n$ , using

$$Z_n = R_a Y_n \quad (5.4.17)$$

where  $Y_n$  is the overlapping series sum in absolute or standardised units.

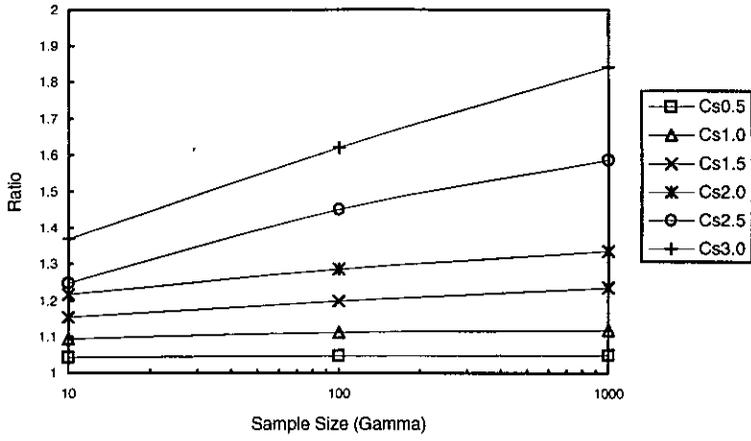


Figure 5.4.1 Ratio  $R_a$  as a function of sample size for different values and  $C_s$  and  $r = 0$

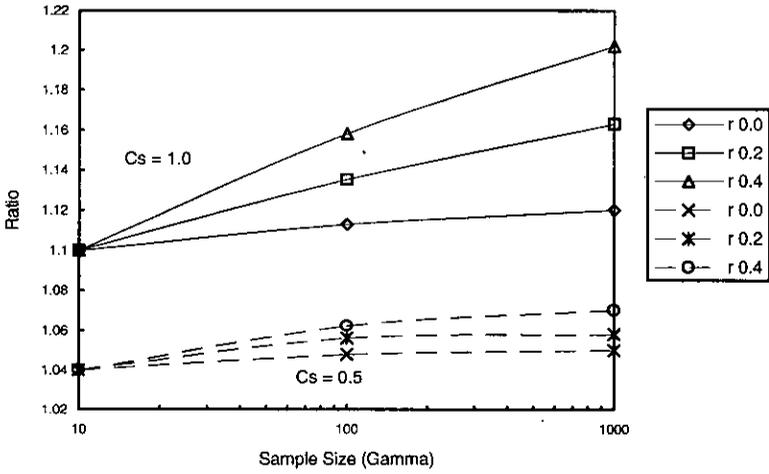


Figure 5.4.2 Ratio  $R_a$  as a function of sample size for different values of  $r$  and  $C_s = 1.0$  and  $0.5$

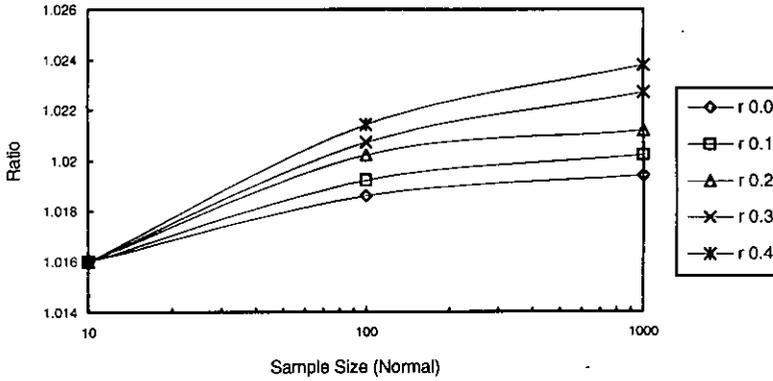


Figure 5.4.3 Ratio  $R_A$  as a function of sample size for Normal data and different values of  $r$

**Probability Calculation and Recurrence Interval**

Use the Normal or Gamma distribution to calculate the probability of non-exceedence,  $p$ .

$$p = \Pr[Z \leq Z_n] \tag{5.4.18}$$

For a Normal distribution, this is done by computing the variate  $(Z - \mu)/\sigma$  and using Normal probability tables to obtain the non-exceedence probability. For the Gamma distribution, use  $\alpha_n$  and  $\beta_n$  and the incomplete Gamma function to obtain  $p$ . Numerical solution of the incomplete Gamma function is given in *Handbook of Mathematical Functions* (Abramowitz and Stengun, 1965) and also available in Excel™ spreadsheet software.

Given the probability of non-exceedence, calculate the event recurrence interval,  $T_n$ , by

$$T_n = n/p \tag{5.4.19}$$

**Example**

Determine the recurrence interval of the lowest 5-year ( $n = 5$ ) consecutive flows for the Colorado River at Lees Ferry, given the following 62 year annual flow series (in  $10^9 m^3$ ).

- 18.1, 21.1, 15.8, 24.2, 14.7, 22.5, 25.8, 16.9, 13.0, 24.7, 25.2, 19.8, 20.9, 14.4, 15.2,
- 16.1, 21.6, 18.1, 24.2, 15.3, 7.7, 18.6, 12.0, 4.9, 12.6, 14.9, 14.8, 14.1, 10.9, 9.3,
- 22.0, 18.2, 14.1, 16.0, 14.5, 10.8, 17.3, 15.9, 18.0, 13.3, 12.2, 22.1, 10.7, 7.6, 8.6,
- 10.7, 23.1, 16.2, 8.7, 10.8, 11.6, 17.8, 1.6, 4.0, 14.3, 9.5, 9.3, 10.7, 11.1, 9.9, 11.3,
- 11.5

Analysis of the data (5.4.1, 5.4.2, 5.4.3) reveals  $C_S = 0.08$  (therefore assume normally distributed data), with  $\mu = 14.9$ ,  $\sigma = 5.5$  and  $r = 0.36$ . The lowest 5-year sum is 38.7.

Standardisation of the data to  $\mu = 10$  and  $\sigma = 1$  results in a series with a corresponding 5-year low flow sum of 43.5

Given  $r = 0.36$ , and accounting for auto-correlation,  $\mu$  of the 5-year sum is

$$\mu_5 = 5 * 10 = 50 \quad (5.4.13)$$

and

$$\sigma_5^2 = R_5 * 5 * 1, \text{ where } R_5 = 1.78, \text{ so } \sigma_5^2 = 8.90 \quad (5.4.14)$$

For  $N = 62$  and  $r = 0.36$ , the value of  $R_a$  from the figures above is 1.020. Thus,

$$Z_5 = 1.020 * 43.5 = 44.37 \quad (5.4.17)$$

Thus, the Normal variate for the 5-year sum is given by  $(Z - \mu_5) / \sigma_5$

$$= (44.37 - 50) / \sqrt{8.90} = -1.89$$

Hence, from Normal distribution tables,  $p = 0.029$ , and the recurrence interval

$$T_5 = 172 \text{ years} \quad (5.4.19)$$

## *References*

- Abramowitz, M. and I.A. Stegun (1965): *Handbook of Mathematical Functions*. Dover Publ. Inc., New York, 1043 pp.
- McMahon, T.A. and R.G. Mein (1986): *River and Reservoir Yield*. Water Resources Publications, Littleton, Colorado, 368pp.

## 5.5 GRAPHICAL TECHNIQUES FOR TREND ANALYSIS

A common problem in the analysis of hydrological data is the detection of trend in long time series. Detection of both sudden and gradual trends over time with and without adjustment for the effects of exogenous variables are best explored visually. A variety of formal tests are available for assessing the *statistical significance* of a trend (see Topic 5.6), though it is considered that any statistical test should be preceded by graphical analyses. Visualisation of the data and its relationship to other time series can provide important insights into the possible causes of the suspected trend, and knowledge of the timing and nature of the trend aids development of a hypothesis that can be subjected to subsequent formal tests of significance.

There are many ways in which trends can be graphically explored. This topic presents just a few techniques that can be used and adapted to a variety of problems.

### *Non-parametric Smoothing*

A variety of functions can be used to identify the underlying trend in noisy data. The simplest smoothing functions are moving averages or medians. With this approach, the data are smoothed by calculating the average (or median) over  $n$  periods around a given time ordinate, and repeating the calculation for every possible value in the data set. Use of the median rather than the mean yields a result less influenced by outliers; the choice of which statistic to use (and indeed the length of the period) is made on a trial and error basis in order to achieve the desired degree of smoothing.

Perhaps the best available smoothing technique is LOWESS (LOcally WEighted Scatterplot Smooth; Cleveland, 1979), which is a computationally intensive approach based on fitting a weighted least squares regression function to every data point. The degree of smoothness can be manually specified in order to alter the influence of outliers. The function is not easily computed, though it is provided with most standard statistical and technical plotting packages.

An example of each of the above smoothers applied to annual streamflows (Table 5.5.1) is shown in Figure 5.5.1.

### *Residual Mass Curve*

A residual mass curve is a plot of the cumulative departures from the mean. Thus, a pronounced peak (or trough) indicates a trend in the data series; a positive slope

indicates periods where the data are greater than the mean, and a negative slope indicates periods where the data are less than the mean. This analysis is most usefully applied to concurrent data series in order to highlight the degree of similarity between trends.

A residual mass curve for the same streamflow set used in Figure 5.5.1 is shown in Figure 5.5.2. The curve clearly reveals that the flows prior to 1960 are greater than the mean, and that generally for the next 30 years the flows are less than the mean annual value. The residual mass curve for the concurrent annual district rainfall values is also shown in Figure 5.5.2. It appears that the apparent trend in streamflow is (not surprisingly!) largely dependent upon climatic variations in rainfall.

### *Accounting for Exogenous Variables*

Residual mass curves and non-parametric smoothers are useful qualitative techniques for comparing the degree of trend in concurrent time series data. However, as seen in the above examples, the influence of exogenous variables (in this case rainfall) confound the nature and identification of the underlying trend.

In order to identify trend more clearly in the variable of interest it is thus necessary to remove the influence of the exogenous variables. This is most easily done by developing a functional relationship between the variable of interest and the exogenous variable, and subjecting the *residuals* of the function to a trend analysis (either by graphical or formal statistical techniques). Thus if it is desired to analyse trends in salinity, it is first necessary to develop a relationship between flow and salinity, and to investigate how the differences between the observed and estimated values of salinity vary through time.

For example, the Tanh function (Topic 6.3) is a powerful, yet simple technique for removing the exogenous influence of rainfall from streamflow. By fitting the Tanh function to the concurrent streamflow and rainfall data used in the above example (as illustrated for another data set in Figure 6.3.2), the differences between observed streamflows and the Tanh function estimates through time highlight the sudden changes in the relationship between streamflow and rainfall that occurred in 1952 and around 1960 (see Figure 5.5.3). A 2 period moving average is used to highlight the trend in Figure 5.5.3, though where the scatter is greater it may be necessary to identify the trend more clearly using one of the non-parametric smoothers mentioned above, or else representing the trend using a residual mass curve.

### *Double Mass Curves*

The double mass curve technique (Searcy and Hardison, 1960) is a simple technique for identifying the timing, magnitude and nature of trends. A double mass curve is constructed by plotting the accumulated values of a suspect time series against one known to be stationary. A break in slope or a gradual change in curvature reveals a

change in the constant of proportionality between the two data, thus indicating the presence of trend.

It is important to note that double mass curves are useful only if the two variables being accumulated are proportional. For example there is little point in constructing a double mass curve between rainfall and streamflow directly as the relationship between the two variables is highly non-linear. Traditionally, this problem has been overcome by plotting the suspect time series against the same variable observed at a nearby site (or more preferably against pooled data); for example, streamflow at one site is plotted against the mean of a group of nearby streamflow records.

The practical use of double mass curves is usually limited as it is generally difficult to find other observations that are known to be stationary over the required period of interest. One way around this problem is to construct a double mass curve based on a *concurrent exogenous variable*, the data for which is usually readily available. In order to ensure that proportionality between the two variables is preserved, it is necessary to base the double mass curve on the observed and estimated values using the same approach as discussed above for the removal of exogenous influences.

For example, instead of constructing a double mass curve between rainfall and streamflow, the curve is constructed using accumulated values of rainfall excess (ie streamflow estimated using the Tanh function) and observed streamflow. The double-mass curve plot for the observed and estimated streamflows used in the above example is shown in Figure 5.5.4, where again the sudden changes in trend are apparent in 1952 and 1960.

This technique is also ideally suited to highlighting gradual trends arising from land use changes. For example, the gradual reduction in yield following bushfires or the introduction of farm dams would result in a curvilinear double-mass curve relationship, without the sharp break points evident in Figure 5.5.4. The method is applicable to any concomitant variables (such as flow and salinity), the only difference being in the form of the transfer function adopted.

## References

- Cleveland, W.S. (1979): Robust locally weighted regression and smoothing scatterplots. *J. Am. Stat. Assoc.*, **74**, pp 829-836.
- Searcy, J. K. and C. H. Hardison (1960). Double mass curves. *U.S. Geol. Surv. Water Supply Paper*. **15 41B**, 66pp.

## Further Reading

Helsel, D.R. and R.M. Hirsch (1992): *Statistical methods in water resources*. Studies in Environmental Science 49, Elsevier, 522pp.

Velleman, P.F. and D.C. Hoaglin (1982): *Applications, basics, and computing of exploratory data analysis*. Duxbury Press, Boston.

Table 5.5.1 Annual flow for Tarago R. (228206) and rainfall data  
(Rainfall District 086) used for examples

Year	Flow (mm)	Rainfall (mm)	Year	Flow (mm)	Rainfall (mm)
1945	190	769	1971	249	930
1946	238	1176	1972	170	657
1947	282	1086	1973	179	993
1948	255	921	1974	281	1024
1949	173	918	1975	264	84
1950	204	889	1976	218	801
1951	280	1125	1977	256	805
1952	525	1345	1978	236	1216
1953	466	1014	1979	172	759
1954	359	913	1980	139	939
1955	275	986	1981	161	874
1956	481	1174	1982	97	628
1957	323	886	1983	136	988
1958	328	992	1984	234	920
1959	268	797	1985	199	953
1960	451	1007	1986	241	927
1961	272	789	1987	175	828
1962	208	897	1988	145	980
1963	237	945	1989	204	988
1964	241	1109	1990	226	925
1965	192	733	1991	236	1044
1966	194	961	1992	303	970
1967	114	638	1993	255	1068
1968	156	990	1994	223	862
1969	170	929	1995	410	1081
1970	269	1165	1996	246	947

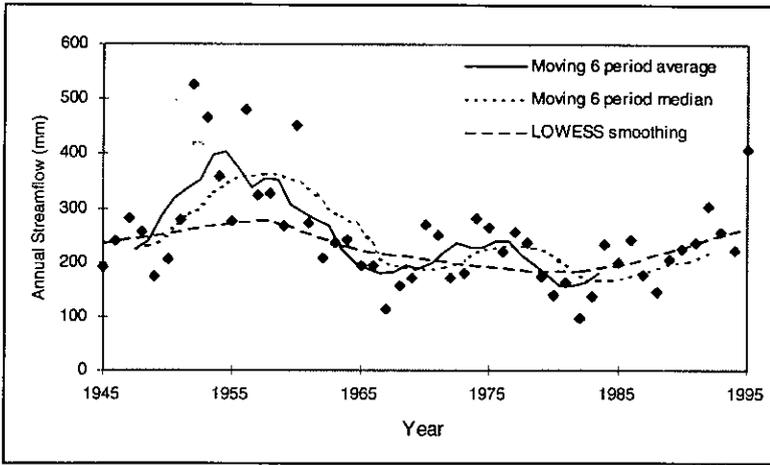


Figure 5.5.1 Application of different smoothing algorithms to annual streamflows recorded at Tarago (Station 228206)

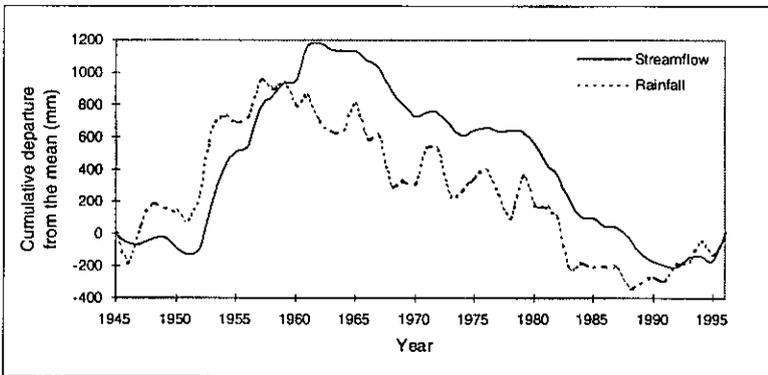


Figure 5.5.2 Residual mass curves derived for annual streamflows recorded at Tarago (Station 228206), and annual rainfalls for Rainfall District 086.

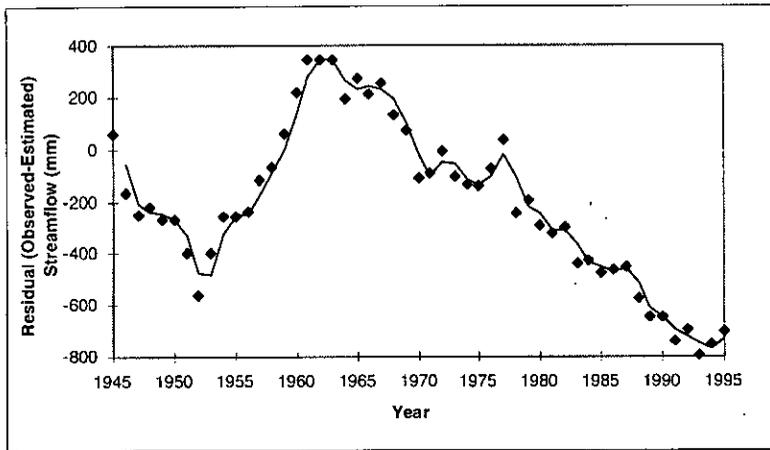


Figure 5.5.3 Application of a 2 period moving average to the differences between observed streamflows recorded at Tarago (Station 228206) and Tanh function estimates based on District 086 rainfalls.

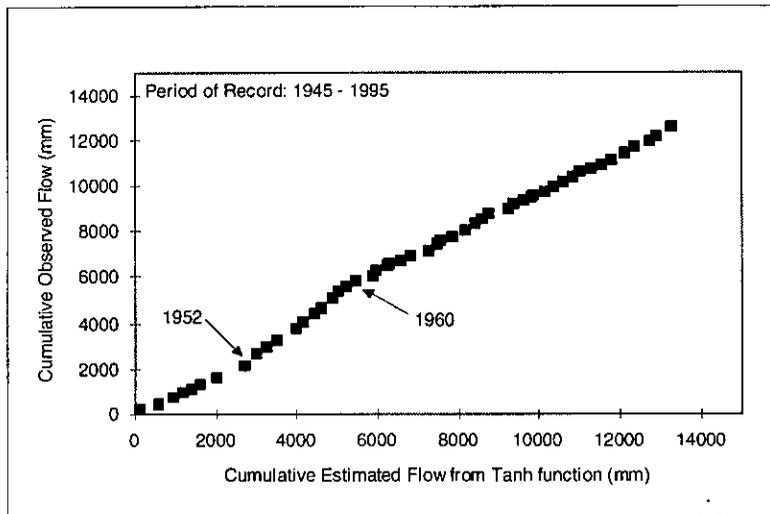


Figure 5.5.4 Double mass curve plot for the observed streamflows recorded at Tarago (Station 228206) and Tanh function estimates based on District 086 rainfalls.

## 5.6 STATISTICAL TESTS FOR RANDOMNESS AND TREND

A common problem in the analysis of hydrological data is the detection of trend in long time series. The graphical techniques in Topic 5.5 should be used to obtain a “feel” for the data and provide some insight into physical explanations for trends. In some instances, there may also be a desire to test whether trends have statistical significance. This topic presents four tests for randomness and five different methods for dealing with the general problem of trend detection. It is important in any use of trend detection methods to ensure that any trend is not a function of exogenous variables i.e. due to some other correlated variable. In the example mentioned above, a decline in stream flow might be detected but one must be sure that this is not due to a correlation with rainfall and a series of dry years coinciding with the post clear-felling period (see also Hirsch et al., 1991). It is also important to ensure the data are not significantly autocorrelated (see below).

Three of the trend detection methods (Cumulative Deviation, Worsley Likelihood Ratio and Distribution Free CUSUM) determine a change point in data, test whether the two subsets are different and indicate which of the two means is higher. The Mann Test indicates the general direction of change in a time series and the Kruskal-Wallis Test detects whether the means of different sub periods are equal.

Because the Mann, Kruskal-Wallis and Distribution-Free CUSUM tests are based on ranks and not the actual data values, they are non-parametric so are not dependent on an underlying statistical distribution of data. The Cumulative Deviation and Worsley Likelihood Ratio tests assume that the data are independent and normally distributed, although can be used if there are only slight departures from normality.

### *Tests for Randomness*

The following four tests can be used to check whether a time series may have resulted from a purely random process.

#### **Autocorrelation Test (Jenkins and Watts, 1968)**

The lag-one autocorrelation coefficient ( $r_k$ ,  $k=1$ ) gives an indication of data independence. The coefficient is given by Equation 5.6.1 for sample size  $n$ .

$$r_k = \frac{\left[ \sum_{i=1}^{n-k} (x_i - \bar{x})(x_{i+k} - \bar{x}) \right]}{\left[ \sum_{i=1}^n (x_i - \bar{x})^2 \right]} \quad (5.6.1)$$

A data series can be tested for short term dependence by checking whether  $r_k$  is significantly different from the expected value  $E(r_1)$ .

$$E(r_1) = -1/n \quad (5.6.2)$$

$$\text{Var}(r_1) = (n^3 - 2n^2 + 2)/[n^2(n^2 - 1)] \quad (5.6.3)$$

$$z\text{-statistic} = [r_1 - E(r_1)] / [\text{Var}(r_1)]^{0.5}$$

If this lies within the critical z-statistic value (95% confidence limits,  $\alpha=0.05$ ,  $z_{\text{crit}} = 1.645$ ), then the hypothesis that the sequence results from a random process is accepted at the level  $\alpha$ .

### Median Crossing Test (Fisz, 1963)

The  $n$  time series values are replaced by 0 if  $x_i < x_{\text{median}}$  and by 1 if  $x_i > x_{\text{median}}$ . If the original sequence had been generated by a purely random process, then  $m$  (the number of times 0 is followed by 1 or 1 is followed by 0) is approximately normally distributed with:

$$\text{mean} = (n - 1)/2 \quad (5.6.4)$$

and

$$\text{variance} = (n - 1)/4 \quad (5.6.5)$$

$$z\text{-statistic} = (m - \text{mean})/(\text{variance})^{0.5}$$

If this lies within the critical z-statistic value (95% confidence limits,  $\alpha=0.05$ ,  $z_{\text{crit}} = 1.645$ ), then the hypothesis that the sequence results from a random process is accepted at the level  $\alpha$ .

### Turning Points Test (Kendall and Stuart, 1976)

This is a similar approach but uses the following criteria for assigning the binary numbers:

If  $x_{i-1} < x_i > x_{i+1}$  or  $x_{i-1} > x_i < x_{i+1}$  then  $x_i$  is assigned 1

otherwise,  $x_i$  is assigned 0.

The number of times 1 appears ( $m^*$ ) is approximately normally distributed with:

$$\text{mean} = 2(n - 2)/3 \quad (5.6.6)$$

and

$$\text{variance} = (16n - 29)/90 \quad (5.6.7)$$

$$z\text{-statistic} = (m^* - \text{mean})/(\text{variance})^{0.5}$$

If this lies within the critical z-statistic value (95% confidence limits,  $\alpha=0.05$ ,  $z_{\text{crit}} = 1.645$ ), then the hypothesis that the sequence results from a random process is accepted at the level  $\alpha$ .

**Rank Difference Test (Meacham, 1968)**

In this test, the actual values are replaced by their relative ranks starting at 1 for the lowest up to n. The statistic U is the sum of the absolute rank differences between successive ranks given by:

$$U = \sum_{i=2}^n |R_i - R_{i-1}| \tag{5.6.8}$$

For large n, U is normally distributed with:

$$\text{mean} = (n + 1)(n - 1)/3 \tag{5.6.9}$$

$$\text{variance} = (n - 2)(n + 1)(4n - 7)/90 \tag{5.6.10}$$

$$z\text{-statistic} = (U - \text{mean})/(\text{variance})^{0.5}$$

If this lies within the critical z-statistic value (95% confidence limits,  $\alpha=0.05$ ,  $z_{\text{crit}} = 1.645$ ), then the hypothesis that the sequence results from a random process is accepted at the level  $\alpha$ .

*Tests for Trend*

**Mann’s Test (Kendall, 1970)**

Given a time series  $(X_1, X_2, X_3, \dots, X_n)$ , Mann’s test statistic tests the null hypothesis  $H_0$ , that the observations are randomly ordered versus the alternative of a monotonic trend over time. Let  $R_1, R_2, R_3, \dots, R_n$  be the ranks of the corresponding X values and define the function  $\text{sgn}(x)$  as follows:

$$\text{sgn}(x) = 1 \text{ for } x > 0, \quad \text{sgn}(x) = 0 \text{ for } x = 0 \text{ and } \text{sgn}(x) = -1 \text{ for } x < 0 \tag{5.6.11}$$

If the null hypothesis is true, the statistic:

$$S = \sum_{i < j} \text{sgn}(R_j - R_i) \tag{5.6.12}$$

has a mean of zero and a variance of:

$$\text{Var}(S) = \frac{n(n-1)(2n+5)}{18} \tag{5.6.13}$$

and is asymptotically normal. The normal z-test statistic is,

$$u(n) = \frac{S}{[\text{Var}(S)]^{0.5}} \tag{5.6.14}$$

The statistic  $u(n)$  can be computed for any values of  $i$  to detect whether there is a trend in the data up to  $i$  at the chosen level of significance using the z-test. A positive value of  $u(n)$  indicates that there is an increasing trend and vice versa.

**Cumulative Deviation Test (Buishand, 1982)**

The purpose of this test is to detect a change in the mean of a time series after  $m$  observations,

$$E(X_i) = \mu \quad i=1,2,3,\dots,m \quad (5.6.15)$$

$$E(X_i) = \mu + \Delta \quad i= m+1, m+2,\dots, n \quad (5.6.16)$$

where  $\mu$  is the mean prior to the change and  $\Delta$  is the change in the mean.

The cumulative deviations from the means are calculated as,

$$S_0^* = 0 \quad S_k^* = \sum_{i=1}^k (X_i - \bar{X}) \quad k=1,2,3,\dots,n \quad (5.6.17)$$

and the rescaled adjusted partial sums are obtained by dividing the  $S_k^*$  values by the standard deviation,

$$S_k^{**} = \frac{S_k^*}{D_x} \quad D_x^2 = \sum_{i=1}^n \frac{(X_i - \bar{X})^2}{n} \quad (5.6.18)$$

The test statistic is,

$$Q = \max |S_k^{**}| \quad 0 \leq k \leq n \quad (5.6.19)$$

and is calculated for each year, with the highest value indicating the change point.

Critical values for  $Q/\sqrt{n}$  are given in Table 5.6.1 after Buishand (1982). If  $S_k^{**}$  is negative, the latter part of the record has a higher mean compared to the earlier part.

Table 5.6.1 Critical values of  $Q/\sqrt{n}$

n	Q/√n at significance level		
	90%	95%	99%
10	1.05	1.14	1.29
20	1.10	1.22	1.42
30	1.12	1.24	1.46
40	1.13	1.26	1.50
50	1.14	1.27	1.52
100	1.17	1.29	1.55
∞	1.22	1.36	1.63

**Worsley Likelihood Ratio Test (Worsley, 1979)**

The Worsley Likelihood Ratio test provides an alternative to the Student's t test when the change point  $m$  is unknown. It is similar to the Cumulative Deviation Test but weights the values of  $S_k^*$  depending on their position in the time series.

$$Z_k^* = [k(n - k)]^{-0.5} S_k^* \tag{5.6.20}$$

$$Z_k^{**} = Z_k^* / D_x \tag{5.6.21}$$

The test statistic is,

$$W = \frac{(n-2)^{0.5} V}{(1 - V^2)^{0.5}} \tag{5.6.22}$$

$$\text{where } V = \max | Z_k^{**} | \tag{5.6.23}$$

Critical values of  $W$  are given in Table 5.6.2 after Worsley (1979).

Table 5.6.2 Critical values of  $W$

n	Critical values of $W$ at significance level		
	90%	95%	99%
3	12.71	25.45	127.32
4	5.34	7.65	17.28
5	4.18	5.39	9.46
6	3.73	4.60	7.17
7	3.48	4.20	6.14
8	3.32	3.95	5.56
9	3.21	3.78	5.19
10	3.14	3.66	4.93
15	2.97	3.36	4.32
20	2.90	3.28	4.13
25	2.89	3.23	3.94
30	2.86	3.19	3.86
35	2.88	3.21	3.87
40	2.88	3.17	3.77
45	2.86	3.18	3.79
50	2.87	3.16	3.79

**Kruskal-Wallis Test (Sneyers, 1975)**

This tests for the equality of sub-period means. Let the time series be divided into  $m$  sub-periods with lengths  $L_j$  ( $j=1,2,\dots,m$ ) and  $R_{ij}$  be the rank of the  $i^{\text{th}}$  observation of the  $j^{\text{th}}$  sub-sample in the ordered complete sample. The test statistic is,

$$XS = 12 \sum_{j=1}^m \frac{\frac{R_j^2}{L_j}}{[n(n+1)] - 3(n+1)} \tag{5.6.24}$$

where  $R_j$  is the total ranks in the  $j^{\text{th}}$  sample (i.e.,  $R_j = \sum_{i=1}^{L_j} R_{ij}$ ). (5.6.25)

Under the null hypothesis of equal sub-period means, the statistic (XS) follows the Chi-square distribution with  $m-1$  degrees of freedom.

This test can be used to test for sub-period variability in annual flows ( $X_i$ ) by doing the ranking on the quantities  $|X_i - \bar{X}|$ .

**Distribution-Free CUSUM Test (McGilchrist and Woodyer, 1975)**

This method determines whether means differ between two parts of a record.

$$V_k = \sum_{i=1}^k \text{sgn}(X_i - X_{\text{median}}) \quad k=1,2,\dots,n \tag{5.6.26}$$

where  $\text{sgn}(x)$  is defined as previously. The distribution of  $V_k$  follows the Kolmogorov-Smirnov two sample statistic ( $KS = (2/n) \max|V_k|$ ) with the critical values of  $V_k$  given by:

90%	$1.22\sqrt{n}$
95%	$1.36\sqrt{n}$
99%	$1.63\sqrt{n}$

A negative value of  $V_k$  indicates that the latter part of the record has a higher mean than the earlier part and vice versa.

**Example**

Annual flow data from the Campaspe River at Ashbourne (406208, area = 33.3 km<sup>2</sup>) for the period 1940 to 1989 (Figure 5.6.1) has been used to compute each of the statistics above. For the Kruskal Wallis test, the sub-periods were chosen to have 10 data points.

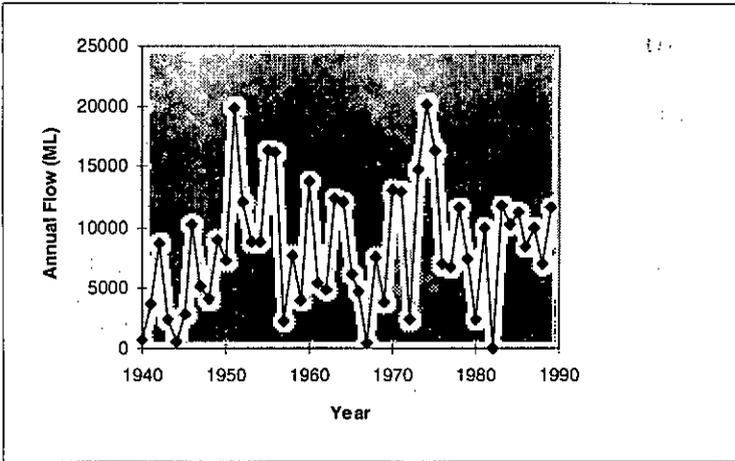


Figure 5.6.1 Annual flow data for the Campaspe River at Ashbourne (Station 406208)

Year	Flow (ML)	Rank	S (5.6.12)	$S_k^{**}$ (5.6.18)	$Z_k^{**}$ (5.6.21)	$V_k$ (5.6.26)
1940	681	4	43	-1.537	-0.220	-1
1941	3661	10	32	-2.475	-0.253	-2
1942	8625	27	-1	-2.416	-0.203	-1
1943	2475	8	34	-3.593	-0.265	-2
1944	573	3	41	-5.152	-0.343	-3
1945	2794	9	34	-6.264	<b>-0.386</b>	-4
1946	10190	33	-9	-5.891	-0.340	-3
1947	5143	16	22	-6.531	-0.356	-4
1948	4139	13	27	-7.373	-0.384	-5
1949	8945	30	-2	-7.250	-0.363	-4
1950	7295	22	11	<b>-7.458</b>	-0.360	-5
1951	19883	49	-36	-5.138	-0.241	-4
1952	12119	39	-17	-4.377	-0.200	-3
1953	8772	28	0	-4.288	-0.191	-2
1954	8848	29	-1	-4.184	-0.183	-1
1955	16309	47	-30	-2.582	-0.111	0
1956	16254	46	-29	-0.990	-0.042	1
1957	2303	5	28	-2.201	-0.092	0
1958	7671	25	1	-2.334	-0.096	-1
1959	3985	12	20	-3.207	-0.131	-2
1960	13742	44	-23	-2.120	-0.086	-1
1961	5333	17	14	-2.722	-0.110	-2
1962	4859	15	15	-3.420	-0.137	-3
1963	12381	41	-16	-2.606	-0.104	-2
1964	12137	40	-15	-1.842	-0.074	-1
1965	6075	18	12	-2.295	-0.092	-2
1966	4669	14	13	-3.031	-0.122	-3
1967	378	2	20	-4.629	-0.187	-4
1968	7507	24	5	-4.794	-0.194	-5
1969	3891	11	14	-5.687	-0.232	<b>-6</b>
1970	13046	43	-13	-4.739	-0.195	-5
1971	12954	42	-12	-3.811	-0.159	-4
1972	2445	6	14	-4.993	-0.211	-5
1973	14759	45	-12	-3.702	-0.159	-4
1974	20200	50	-15	-1.318	-0.058	-3
1975	16331	48	-14	0.290	0.013	-2
1976	6922	21	5	0.006	0.000	-3
1977	6739	19	8	-0.313	-0.015	-4
1978	11629	36	-7	0.349	0.017	-3
1979	7351	23	4	0.152	0.000	-4
1980	2445	7	7	-1.031	-0.054	-5
1981	9960	31	2	-0.703	-0.038	-4
1982	10	1	7	-2.375	-0.137	-5
1983	11786	38	-6	-1.681	-0.103	-4
1984	10214	34	-1	-1.303	-0.087	-3
1985	11216	35	-2	-0.723	-0.053	-2
1986	8393	26	1	-0.711	-0.060	-1
1987	10005	32	0	-0.375	-0.038	0
1988	6896	20	1	-0.663	-0.095	-1
1989	11632	37	0	0.000	0.000	0

Summary Statistics: mean = 8331; median = 8000; std. deviation = 5028;  $C_v = 0.604$ ;  
Skew = 0.365

Autocorrelation Test

Statistic ( $r_1$ )	0.189
Expected mean	-0.02
Standard deviation	0.138
z-statistic	1.516
Conclusion	<b>Nothing to suggest data do not come from a random process (at 5% level).</b>

Median Crossing Test

Statistic	23
Expected mean	24.5
Standard deviation	3.5
z-statistic	0.429
Conclusion	<b>Nothing to suggest data do not come from a random process (at 10% level).</b>

Turning Points Test

Statistic	32
Expected mean	32
Standard deviation	2.9
z-statistic	0.000
Conclusion	<b>Nothing to suggest data do not come from a random process (at 10% level).</b>

Rank Difference Test

Statistic	755
Expected mean	833
Standard deviation	72.5
z-statistic	1.077
Conclusion	<b>Nothing to suggest data do not come from a random process (at 10% level).</b>

Mann Test - calculated using the whole data set

Total S score	174
Standard Deviation	119.6
z-statistic	1.455
Conclusion	<b>Data shows an increasing trend, statistically significant at 10% level</b>

Cumulative Deviation Test

Q/ $\sqrt{n}$  1.055

Year of change 1950

Conclusion **Mean of 1950-89 is > 1940-49 but not statistically significant at 10% level**

Worsley Likelihood Ratio Test

W statistic 2.895

Year of change 1945

Conclusion **Mean of 1945-89 is > 1940-44 and is statistically significant at 10% level.**

Kruskal-Wallis Test

K-W statistic 9.21

Conclusion **Flows in certain 10 year sub-periods are different from flows in other periods, statistically significant at 5% level.**

Distribution Free CUSUM Test

Maximum deviation 6

Year of change 1969

Critical value at 10% =  $1.22n^{0.5} = 8.6$

Conclusion **Flows in later years are higher than earlier years but the difference is not statistically significant at 10% level.**

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## Further Reading

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## 6.1 RATING CURVE EXTRAPOLATION

Rating curve extrapolation is generally used for estimating the stage-discharge relationship for flows higher than those observed, although lower-end extension is also of interest. Common techniques involve either graphical extrapolation or one of the mathematical formulations given here. If large extrapolations are to be made, then simple mathematical extrapolation may be inadequate, and any estimations should be checked with experienced field hydrographers, giving close attention to stage-related changes in cross sectional geometry and controls.

In any extrapolation, it is always worth graphing the data to get a visual "feel" for the stream behaviour and ensure that the extrapolation makes physical sense. Graphical representations of velocity, area and flow for the gaugings, proposed ratings and proposed extrapolations for both high and low flows should be explored using combinations of linear-linear, linear-log and log-log plots with surveyed cross sections and surveyed cease to flow levels.

Three commonly used techniques based on different mathematical functions are discussed in this topic. These are Logarithmic Extension, Conveyance-Manning/Chester and Discharge<sup>0.4</sup> (Mosley and McKerchar, 1993; HydroTechnology, 1994; Brian Chester, pers. comm.).

### *Logarithmic Extension*

Logarithmic extension is performed through regression of stage against discharge on logarithmic scales. The assumed relationship between discharge and stage is given by:

$$Q = \alpha h^m \quad (6.1.1)$$

or  $\log Q = m (\log h) + \log \alpha \quad (6.1.2)$

where

Q	discharge
h	stage above zero flow level
$\alpha$	constant
m	constant exponent

Extrapolation is performed by regressing  $\log Q$  against  $\log h$  and determining the values of  $\alpha$  and  $m$ . Small extensions of the relationship can then be performed for desired stages or discharges, provided there are no control changes in the channel or higher non-linearities due to the occurrence of flow above bankfull discharge.

## Conveyance - Manning / Chester

The conveyance method relates discharge to the product of a discharge coefficient and a conveyance parameter related to channel cross-sectional area and hydraulic radius. The approaches are based upon Manning's Equation 6.1.3 and a formulation by Brian Chester (Equation 6.1.4) of the Water Authority of Western Australia, respectively.

$$Q = C_m AR^{2/3} \quad (6.1.3)$$

where

$C_m$  discharge coefficient  
 $A$  cross-sectional area [m<sup>2</sup>]  
 $R$  hydraulic radius [m]

$$Q = C_d Ah^{0.5} \quad (6.1.4)$$

where

$C_d$  discharge coefficient  
 $A$  cross-sectional area [m<sup>2</sup>]  
 $h$  stage [m]

For extrapolation, the discharge coefficient is found by plotting measured  $Q$  against the particular conveyance parameter ( $AR^{2/3}$  or  $Ah^{0.5}$ ) calculated from site measurements at the corresponding stages. It is then possible to compute  $Q$  for appropriate values of  $A$  and  $R$ , or  $h$ .

## Discharge<sup>0.4</sup>

The method arose from work in Western Australia, where large discharge ranges occur. In this method linear stage is regressed against discharge raised to the power of 0.4.

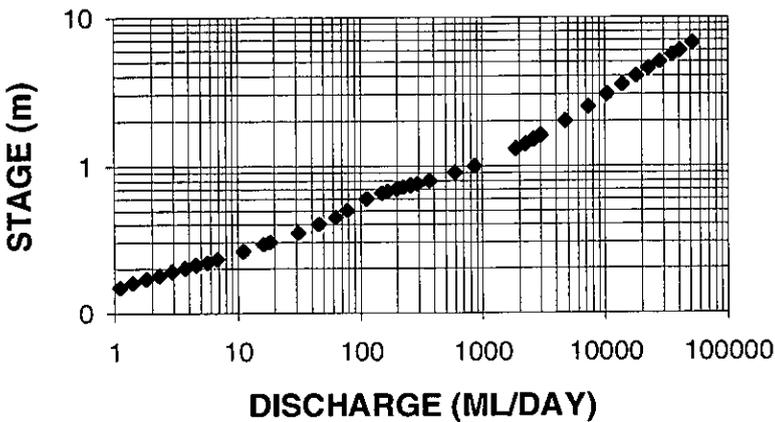
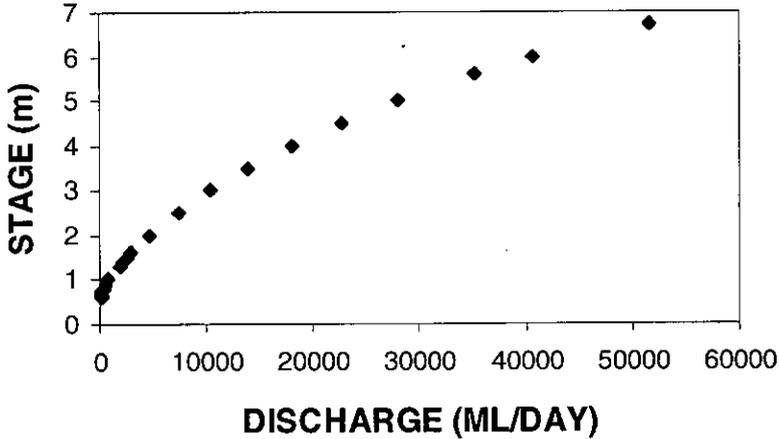
$$h = \beta Q^{0.4} \quad (6.1.5)$$

Plotting is done on linear (arithmetic) paper, thereby removing the scale disadvantages associated with plotting values on logarithmic paper. Extrapolation is performed by regression analysis of the stage-discharge relationship to determine the value of  $\beta$ . It is then a simple matter to evaluate the desired discharge (or stage) from a particular stage (or discharge).

**Example**

Estimate the discharge for a stage of 6.8 m in Freestone Creek at Briagolong (225218) using logarithmic extension. The rating curve for this station is given below, along with a plot of  $\log h$  against  $\log Q$ .

Note that by convention,  $Q$  is plotted on the horizontal axis although the regressions are performed as indicated in the equations.



Examination of the log-log plot indicates slope changes at approximately 100 and 1000  $\text{MLday}^{-1}$ , possibly due to changes in control conditions. Linear regression in the log

domain of all data above  $1000 \text{ MLday}^{-1}$  was undertaken, yielding a relationship of the form of (6.1.2):

$$\log Q = 2.792 + 2.471(\log h),$$

Thus  $\log Q = 4.849 \Rightarrow Q = 70700 \text{ ML day}^{-1}$

## *References*

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- Mosley, M.P. and A.I. McKerchar (1993): *Streamflow*. In Maidment, D.R. (Ed) *Handbook of Hydrology*. McGraw-Hill Inc.

## *Further Reading*

- Hersch, R.W. (1985): *Streamflow Measurement*. Elsevier Applied Science, London.

## 6.2 BASE FLOW SEPARATION AND THE BASE FLOW INDEX

### *Digital filters for the separation of hydrographs into quick and base flow*

It is often desirable to split the flow measured at a gauging station into components representing stormflow and baseflow response, often thought of as surface and subsurface runoff. While this sounds simple, in practice it is not and has been described as "that fascinating arena of fancy and speculation" (Appleby, 1970). Despite the arguments regarding the physical reality of various methods, the practical problem remains and standard methods are in wide use (e.g. Pilgrim and Cordery, 1993). Most of these are graphical and require various subjective decisions to be made such as the point at which surface runoff ceases and the actual shape of the base flow hydrograph. In addition, these methods are not well suited for use with computers.

The methods described in this topic are based on digital filters. Early work (O'Loughlin et al., 1982; Chapman, 1987; Nathan and McMahon, 1990) was based on a filter commonly used for signal processing (Lyne and Hollick, 1979) and has been shown to yield similar results to conventional methods (Nathan and McMahon, 1990) for specific values of the parameters. It is also included as a subroutine in the HYDSYS package. More recently, two equations have been presented by Chapman and Maxwell (1996) which have a more attractive theoretical basis. All three techniques are described in this topic along with some suggestions as to appropriate applications.

**It must be stressed, that the resulting "quickflow" and "base flow" from any of the methods should not be regarded as the true amounts of surface and subsurface flow from the catchment. The methods are simply consistent, robust and expeditious techniques for numerically separating streamflow data in rapid and slow response. Only when additional information is available such as from tracer studies can physical interpretations be put on the filtered responses.**

#### *Method 1 (Chapman and Maxwell, 1996)*

This method is suited to the separation of "quickflow" and "baseflow" from long periods of flow record. It requires an estimate of the constant ( $k$ ) which can be considered as the recession constant of the hydrograph. Standard hydrology texts provide a number of ways to calculate recession constants (see also Nathan and McMahon, 1990).

$$q_{b(i)} = \frac{k}{2-k} q_b(i-1) + \frac{1-k}{2-k} q(i) \quad (6.2.1)$$

subject to  $q_{b(i)} \leq q(i)$

where

- $q_{b(i)}$  filtered baseflow response for the  $i^{\text{th}}$  sampling instant
- $q(i)$  original streamflow for the  $i^{\text{th}}$  sampling instant
- $k$  filter parameter given by the recession constant (several methods for computation are given in Nathan and McMahon, 1990)

The filter is run as a single pass through the data.

### *Method 2 (Boughton, 1993; Chapman and Maxwell, 1996)*

This is a more flexible digital filter that has been used to match flow path separation data for storm events from tracer studies. The equation is:

$$q_{b(i)} = \frac{k}{1+C} q_{b(i-1)} + \frac{C}{1+C} q(i) \quad (6.2.2)$$

subject to  $q_{b(i)} \leq q(i)$

where

- $C$  parameter that enables the shape of the separation to be altered

Again it is applied as a single pass through the data. It is particularly useful when additional data are available on actual flow path separation (e.g., tracer information). In this case, Equation 6.2.2 may be used to fit a curve to the data by adjusting the parameter  $C$ . It should be noted that flow path separation using tracers does not necessarily provide information on the stormflow / baseflow components of total flow. Tracer studies generally show the proportion of runoff that has flow through, rather than over, the soil on its way to the stream. This can be useful for water quality modelling but it must be kept in mind that some of these flow paths can result in very rapid response

### *Method 3 - Lyne and Hollick filter (Nathan and McMahon, 1990)*

The methods above have a better theoretical basis than the Lyne and Hollick filter (Chapman and Maxwell, 1996; Chapman, 1991) but the latter has been widely applied to daily data and there is a body of regionalised information available, based on its use. It is presented here in case the reader wishes to utilise some of that existing information. The equation is as follows:

$$q_f(i) = \alpha q_f(i-1) + (q(i) - q(i-1)) \frac{(1 + \alpha)}{2} \quad (6.2.3)$$

for  $q_f(i) \geq 0$

where

- $q_f(i)$  filtered quick flow response for the  $i^{\text{th}}$  sampling instant
- $q(i)$  original streamflow for the  $i^{\text{th}}$  sampling instant
- $\alpha$  filter parameter for which a value of 0.925 is recommended for daily data (see Nathan and McMahon, 1990)

Base flow ( $q_b$ ) is therefore  $q_b = q - q_f$

When coding the algorithm into a spreadsheet or computer program, a conditional equation should be used where if the computed value of  $q_f$  is less than zero,  $q_b$  is set to  $q$ , otherwise  $q_b$  equals  $q - q_f$ . The filter should be applied in three passes for use as described in Nathan and McMahon (1990). The first and third are “forward” passes using Equation 6.2.3 directly. The second is a “backward” pass using  $i+1$  in place of  $i-1$  in Equation 6.2.3. In the first pass,  $q(i)$  is the measured streamflow, in the second pass  $q(i)$  is the computed baseflows from the first pass and in the third pass,  $q(i)$  is the computed baseflow from the second pass. These passes act to smooth the data.

### Base Flow Index

Base Flow Index is defined as the volume of base flow divided by the total volume of stream flow and is a parameter in some models. Once the base flow has been computed as above, it is a trivial exercise to compute the BFI.

#### Example

Equation 6.2.1 was coded in Excel™ using  $k = 0.95$  and Equation 6.2.2 using  $k = 0.95$  and  $C=0.15$ . These were applied in a single pass to the daily flow data from the Bass River at Loch between 30/6/74 to 4/9/74. Equation 6.2.3 with  $\alpha=0.925$  was applied to the same data in three passes as described above. The flow data are shown below and the resulting separation in Figure 6.2.1.

Daily flows for Bass River at Loch: 5, 7, 108, 117, 57, 36, 26, 95, 1169, 308, 144, 89, 62, 48, 40, 35, 73, 82, 342, 393, 310, 275, 260, 245, 256, 141, 119, 934, 382, 158, 96, 122, 103, 83, 67, 148, 366, 161, 119, 82, 330, 294, 261, 266, 153, 247, 703, 498, 286, 163, 124, 85, 94, 81, 62, 47, 37, 30, 26, 24, 24, 22, 21, 20, 19, 18, 18

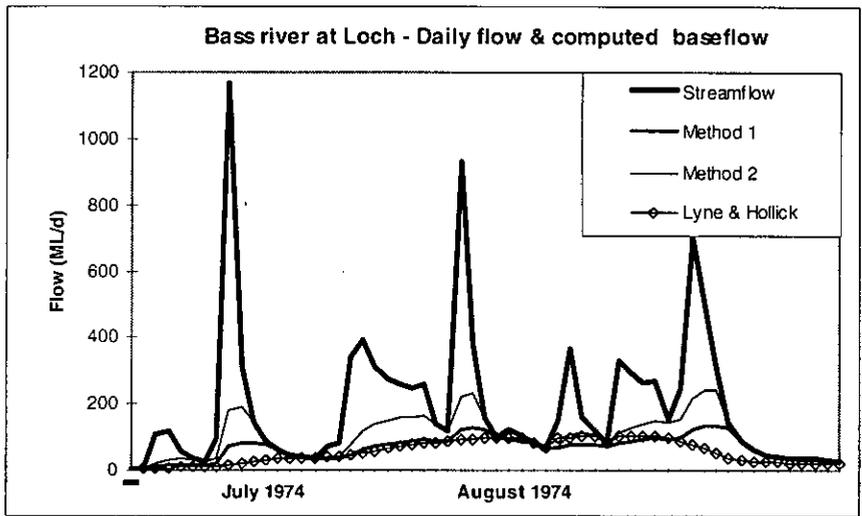


Figure 6.2.1 Baseflow separation using different methods

## References

- Appleby, V.C. (1970): Recession and the baseflow problem. *Water Resour. Res.*, **6(5)**, pp.1398-1403.
- Boughton, W. C. (1993) A hydrograph-based model for estimating water yield of ungauged catchments. *Institution of Engineers Australia, I.E. Aust. Natl Conf. Publ. No. 93/14*, pp. 317-324.
- Chapman, T.G. (1987): Unit hydrograph identification using only streamflow data. *Trans. Inst. Eng. Aust.* **CE29**, pp. 187-191.
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- Chapman, T.G. and A.I. Maxwell (1996): Baseflow separation - comparison of numerical methods with tracer experiments. *I.E. Aust. Natl Conf. Publ.* **96/05**, pp. 539-545.
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- O'Loughlin, E.M., Cheney, N.P. and J. Burns. (1982): The Bushrangers experiment: Hydrological response of a Eucalypt catchment to fire. *I.E. Aust. Natl Conf. Publ.* **82/6**, pp. 132-138.
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- Pilgrim, D.H. and I. Cordery. (1993): *Flood Runoff*. Chapter 9 in Maidment, D.R. (Ed) *Handbook of Hydrology*. McGraw-Hill Inc.

## 6.3 THE TANH FUNCTION FOR INFILLING RAINFALL-RUNOFF DATA

The Tanh rainfall-runoff approach provides an effective site-based relationship that is primarily used for infilling monthly or annual runoff values on the basis of measured rainfall. Tanh is a standard hyperbolic function and was used by Boughton (1966) as a simple rainfall-runoff relationship. Similar forms to this equation are the basis of rainfall - runoff relationships such as the USDA Curve Number, and those of Budyko (1977) and Nemeč and Rodier (1979).

### Calculation

$$Q = (P-L) - F \tanh[(P-L)/F] \quad (6.3.1)$$

where

Q	runoff [mm]
P	rainfall [mm]
L	notional loss [mm]
F	notional infiltration [mm]

Equation 6.3.1 can be applied to any data but should be used for data where average storage of soil water is approximately constant i.e. where the notional loss and infiltration might be expected to be similar. Annual data satisfies this requirement but monthly data will need to be separated into data for each month or at least season and a different L and F derived for each month's (or season's) set.

### Determination of F and L

The values of the notional loss, L, and infiltration, F, are determined by plotting monthly flow sets, seasonal flow sets or annual flows against the associated rainfall. A preliminary value of L is chosen from the data and F fitted either by trial and error or with a curve fitting technique. Similarly the preliminary estimate of L can be changed to improve the fit. It is often simplest to just plot the data in a spreadsheet and visually fit the parameters.

### Example

A set of annual rainfall runoff data for Jimmy Creek at Jimmy Creek are available for a 21 year period. However, one year (rainfall = 450 mm) is missing a runoff value. Estimate the runoff for that year.

Steps:  
First we plot the data.

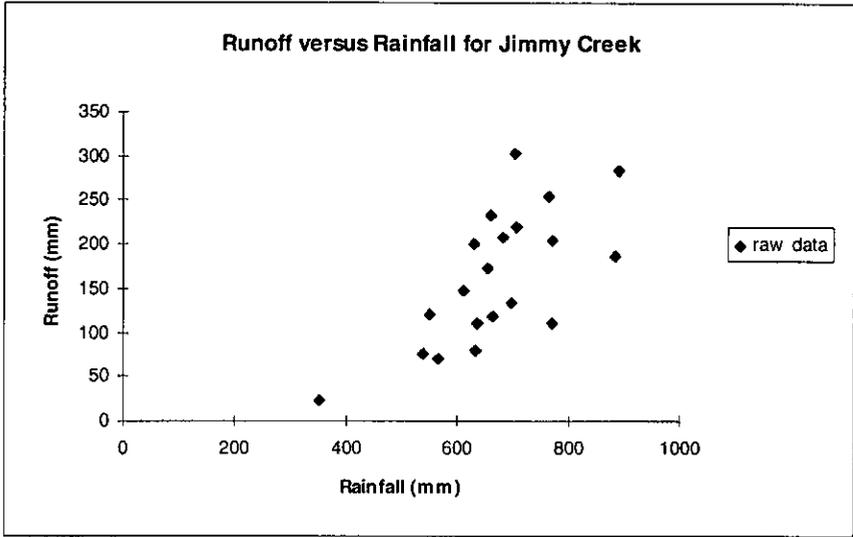


Figure 6.3.2 Runoff versus rainfall, Jimmy Creek

Inspection of the chart shows that little or no runoff occurs for rainfall values below approximately 300 mm. Thus, 300 mm will be selected as the notional loss. By trial and error, an F value of 250 gives a fit as shown in Figure 6.3.2.

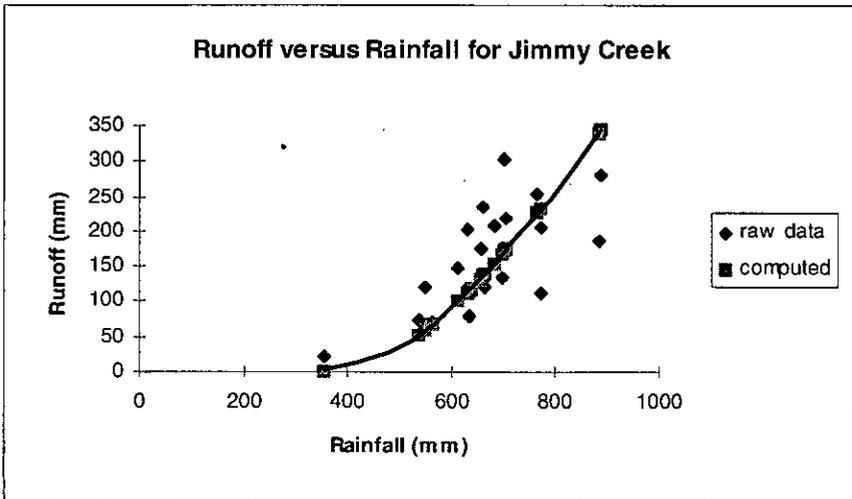


Figure 6.3.2 Data plotted with computed Tanh curve

Thus for the year of missing flow, with a rainfall of 450 Equation 6.3.1 leads to:

$$\begin{aligned} Q &= (450-300) - 250.\tanh((450-300)/250) \\ &= 16 \text{ mm} \end{aligned}$$

## References

- Boughton, W.C. (1966): A mathematical model for relating runoff to rainfall with daily data. *Civil Engineering Transactions, I.E. Aust*, **CE8 (1)**, April, 1966. pp. 83-97.
- Budyko, M.I. (1977): *Global'naja Ekologija*. Mysl Publishers, Moscow.
- Nemec, J and J.A. Rodier (1979): Streamflow characteristics in areas of low precipitation (with special reference to high and low flows). In *The Hydrology of Areas of Low Precipitation. IAHS-AISH Publ.* **128**, pp. 128-140.

## 6.4 EXTENDING A SHORT FLOW RECORD

At times it is desirable to gain an estimation of flow for a catchment for which there is no gauging data, but for which a short period of flow measurement can be undertaken. An approach to this estimation lies in developing a relationship between flow in the candidate catchment and flow in a nearby, gauged catchment where rainfall patterns and streamflow response will be similar. Over the short period of gauging time that is available, concurrent flow measurements in the candidate catchment and the gauged catchment are taken. These are used to obtain a low flow relationship, indicative of the relative baseflow characteristics of the two catchments. For higher flows, which are assumed to be independent of catchment substrata conditions, the ratio of the flows is assumed to be equal to the ratio of the catchment areas to the power of  $b$ . This exponent varies widely and reported values range from 0.5 to 0.85 (e.g., Alexander, 1971; Boyd, 1978; McMahon, 1982). It depends mainly on the combined effects of the reduction in average rainfall intensity with increasing catchment area and the effect of storage in the catchment. If data from high flow events are available,  $b$  can be derived but if not, a value of 0.7 may be used.

Thus the relationship developed will appear as in Figure 6.4.1, allowing the estimation of the flow for the candidate catchment by relating it to flow in the gauged catchment.

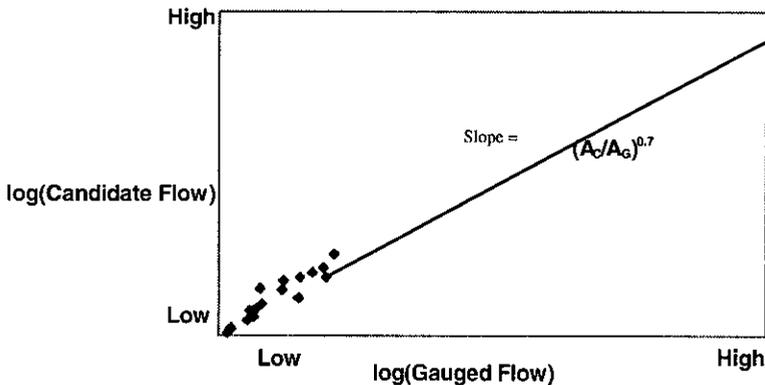


Figure 6.4.1 Log-Log plot of gauged versus candidate flows

### Calculations

- 1) Obtain a series of concurrent flow values for the candidate catchment and the adjacent, gauged catchment, both of which should be unregulated.
- 2) Plot the flows against each other on log-log paper and observe the relationship between the lower flow characteristics of the two catchments in the low flow range.
- 3) For higher flows, calculate the multiplier function,  $F$ , by

$$F = (A_c/A_g)^{0.7} \quad (6.4.1)$$

where

$A_C$  catchment area of the unregulated candidate catchment [ $\text{km}^2$ ]

$A_G$  catchment area of the unregulated gauged catchment [ $\text{km}^2$ ]

- 4) Prepare a graph for estimating flows for the candidate catchment by using the measured low flows and high flows computed from  $Q_C = F Q_G$ , with a smooth curve fitted in the transition range between the high and low flows.

Note: The multiplier factor  $F$  (eqn. 6.4.1) is also useful for estimating flow values at sites away from a gauging station but on the same stream. In this case, the numerator becomes the area upstream of the site of interest while the denominator remains the area of the gauged catchment.

### Example

Develop a relationship for estimating flow for an ungauged  $21 \text{ km}^2$  catchment adjacent to an  $84 \text{ km}^2$  gauged catchment. Streamflow gauging was undertaken on a number of occasions resulting in the data shown in Figure 6.4.2.

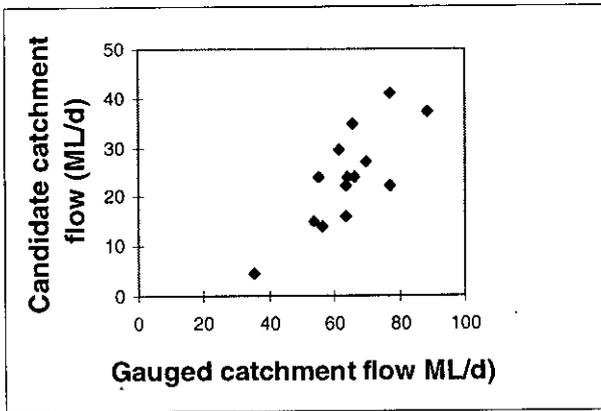


Figure 6.4.2 Concurrent measurements of streamflow for the gauged and candidate catchments

The multiplier factor,  $F$ , is (6.4.1):

$$F = (21/84)^{0.7} = 0.38$$

Thus the relationship for the two catchments is given in Figure 6.4.3. This relationship allows flows to be estimated for the ungauged catchment from those recorded in the gauged catchment.

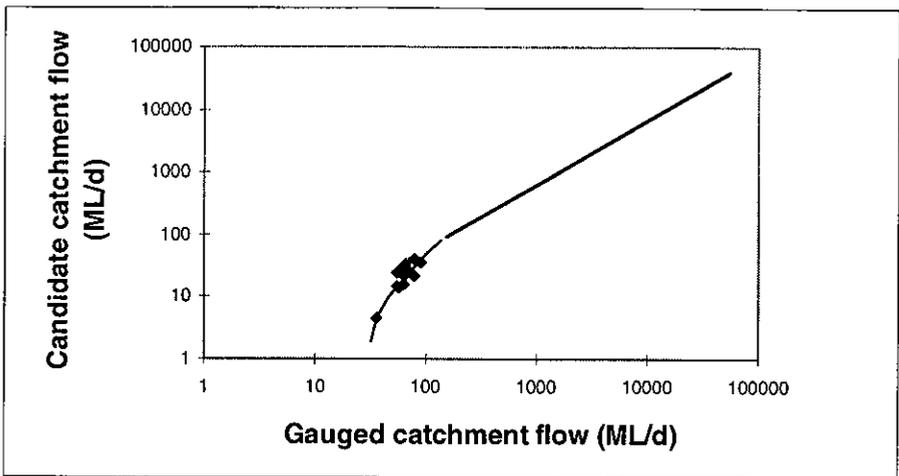


Figure 6.4.3 Flow data with transition curve and line of slope F

## References

- Alexander, G.N. (1972): Effect of catchment area on flood magnitude. *Jour. of Hydrology*, **16**, pp. 225-240.
- Boyd, M.J. (1978): Regional flood frequency data for NSW streams. *I.E. Aust. Civ. Eng. Trans.*, **CE20(1)**, pp. 88-95.
- McMahon, T.A. (1982): World hydrology: Does Australia fit? *Hydrology and Water Resources Symposium, I.E. Aust. Nat. Conf. Publ. No. 82/3*, pp. 1-7.

## 7.1 USING ANDREWS CURVES FOR REGIONALISATION

### *An Alternative Method for Defining Similar Behaviour*

It is commonly desirable to transfer relationships derived from a data-rich catchment to areas that may be considered "hydrologically similar". This is often assumed to mean geographically similar, however many detailed regionalisation studies have indicated that geographical proximity is not a good determinant of hydrological similarity. Indeed the delineation of homogeneous hydrological regions is a key challenge for hydrology.

A graphical method for assessing similarity with respect to chosen measurable parameters is the use of Andrews' Fourier plots (Andrews, 1972; Nathan and McMahon, 1990). These curves allow the multi-dimensional relationship between catchment variables to be represented as a two dimensional curve which is visually inspected for similarity. An example of their use is given in Topic 7.2.

It should be noted that this method provides no special insight into the physical function of the catchments and is simply another tool like cluster or principal components analysis.

Andrews' curves are defined as follows:

$$f(t) = \frac{x_1}{\sqrt{2}} + x_2 \sin(t) + x_3 \cos(t) + x_4 \sin(2t) + x_5 \cos(2t) + x_6 \sin(3t) + x_7 \cos(3t) + \dots$$

where

$f(t)$  two dimensional curve plotted from  $-\pi$  to  $+\pi$

$x_1, x_2$  attributes used to characterise the catchment

Note that the attributes should be standardised to be within the same order of magnitude by, for example, subtracting the mean and dividing by the standard deviation. The first few variables dominate low frequency components while the later variables are associated with high frequency variation. Since low frequency response is more readily seen, it is advisable to make  $x_1$  the most important variable,  $x_2$  the second most important and so on. The method for use of Andrews' curves in assessing catchment similarity is given in Nathan (1993) and summarised below.

- (i) Select a number of gauged catchments thought to be hydrologically similar to the ungauged catchment of interest with respect to the particular response of interest i.e. low flows, peak discharges etc. This may be done intuitively or via a method such as cluster analysis.
- (ii) Measure a range of physical and meteorological characteristics for each selected catchment (e.g., area, mean elevation, % forest cover, mainstream length, mainstream slope, stream density, mean annual rainfall, coefficient of variation of monthly rainfall).

- (iii) Derive stepwise multiple regression equations to identify characteristics most useful in explaining the observed variation in the streamflow characteristics.
- (iv) Plot Andrews curves for each catchment using standardised characteristics from (iii).
- (v) Plot the Andrews curve for the ungauged catchment, overlay it on the curves from (iv) and choose the catchment most similar (i.e. the closest curve). If none are close, more gauged catchments will need to be included.

Success of the method requires the chosen catchment variables to be useful predictors of the flow characteristic of interest. There is a degree of subjectivity in the approach which must be considered in interpreting the results.

### *References*

- Andrews, D.F. (1972): Plots of high dimensional data. *Biometrics*, **28**, pp. 128-136.
- Nathan, R.J. (1993): On the assessment of catchment similarity for the transposition of hydrologic indices. *Hydrology and Water Resources Symposium, I.E. Aust. Natl. Conf. Publication*, **93/14**, pp. 93-98.
- Nathan, R.J. and T.A. McMahon (1990): Identification of homogeneous regions for the purposes of regionalisation. *Jour. of Hydrology*, **21**, pp. 217-238.

### *Further Reading*

- Nathan, R.J. and P.E. Weinmann (1993): *Low Flow Atlas for Victorian Streams*. Long Term Planning Guidelines Background Report - L3, Department of Conservation and Natural Resources, Victoria.

## 7.2 AN IMPROVED METHOD FOR ESTIMATING RORB $k_C$

### *Prediction equations for $k_C$ based on catchment characteristics*

ARR87 includes a number of prediction equations for RORB  $k_C$  values however have a high degree of uncertainty. This topic presents a method to determine  $k_C$  (for  $m=0.8$ ) that reduces the uncertainty (Dyer et al., 1995). It was derived from the analysis of 72 catchments from around Australia and is based on the premise that catchment characteristics rather than geographical proximity is a more appropriate basis for grouping catchments with respect to their  $k_C$  value. The method described requires the measurement of a number of catchment characteristics made from 1:100,000 topographic maps. The method is designed for use with RORB models that use the initial loss-proportional loss model and time delay being a function of reach length (L). The results can be used with initial loss-continuing loss models using the adjustment equation provided. The resulting  $k_C$  can be used in models where time delay is a function of  $LS^{-0.5}$  (where S is the stream slope) but a model using L will also need to be run for application of the method itself.

### *Method*

- (i) Prepare the RORB catchment model using reach length as a predictor of time delay.
- (ii) Determine the values of the eight catchment characteristics in Table 7.2.1.

Table 7.2.1 Catchment parameters for Andrews curve analysis

longdig	longitude of gauging station in decimal format
$d_{av}$	RORB model parameter [km]
for	fraction of forest cover (medium + dense as defined on 1:100,000 map series) (min value = 0.01)
pem	ratio of median annual rainfall to class A pan evaporation
cd2	number of code 2 conceptual storages in RORB
rdy	average number of rain days in a year
mxel	maximum elevation of the catchment above AHD [m]
rla	ratio of length of reaches modelled in the RORB model [km] to catchment area [km <sup>2</sup> ]

(iii) Use the following equations to convert the parameters obtained from step (ii).

$$x_1 = (\ln(\text{longdig}) - 4.95) \times 2.24 \quad (7.2.1)$$

$$x_2 = (\ln(\text{pem}) + 0.49) \times 3.51 \quad (7.2.2)$$

$$x_3 = (\ln(d_{av}) - 2.92) \times 2.05 \quad (7.2.3)$$

$$x_4 = (\ln(\text{for}) + 1.43) \times 0.79 \quad (7.2.4)$$

$$x_5 = (\ln(\text{cd}2) - 1.62) \times 0.60 \quad (7.2.5)$$

$$x_6 = (\ln(\text{rdy}) - 4.60) \times 2.09 \quad (7.2.6)$$

$$x_7 = (\ln(\text{mxel}) - 6.58) \times 2.12 \quad (7.2.7)$$

$$x_8 = (\ln(\text{rla}) + 1.24) \times 2.10 \quad (7.2.8)$$

(iii) Plot the Andrews curve (described in Topic 7.1) for the catchment and compare it to the curves in Figure 7.2.1. Note that the curves in Figure 7.2.1 have been drawn using the data in Table 7.2.3 and a simple curve smoothing option in Excel™. If there is a curve which matches reasonably well then use the appropriate prediction equation from Table 7.2.2 to calculate  $c_{0.8}$ . If none of the type curves seem appropriate, the following general prediction equation may be used however the error will be no better than for those in ARR87.

$$c_{0.8} = 39.4 d_{av}^{-0.25} \text{for}^{0.15} \text{mxel}^{-0.36} \text{pem}^{0.57} \quad (7.2.9)$$

(iv) If the RORB model is to use an initial loss-continuing loss function modify the value of  $c_{0.8}$  as follows:

$$c_{0.8} = 1.08c_{0.8} + 0.14 \quad (7.2.10)$$

(v) Calculate  $k_c$  (for  $m=0.8$ ) from:

$$k_c = c_{0.8} d_{av} \quad (7.2.11)$$

Table 7.2.2 Prediction equations for  $c_{0.8}$  for each group

GROUP	PREDICTION EQUATION
1	$c_{0.8} = 0.405 pem^{-1.82} lrat^{0.18}$
2	$c_{0.8} = 139 minel^{0.27} rr^{-0.50} nn^{-0.38} sa^{-1.46}$
3	$c_{0.8} = 0.445 d_{av}^{-0.73} rla^{-0.90} strm^{-2.22} nn^{-0.70} medrn^{0.88}$
4	$c_{0.8} = 1.04 rlm^{0.77} lnn^{-0.36}$
5	$c_{0.8} = 0.232 rlen^{-0.34} rrd^{1.13} sa^{1.13} cd2^{-1.45}$
6	$c_{0.8} = 20.6 circ^{0.79} for^{1.38} rlt^{-0.28} lnms^{0.32}$
7	$c_{0.8} = 11.1 circ^{1.08} pe^{0.74} sa^{0.79}$

**where**

circ      circularity given by area/perimeter [km]

lnms      ratio of length of streams of order strm-1 (lnn) to mainstream length (msl) using Strahler ordering system (see Gordon et al., 1992).

lnn      length of streams of order strm-1 [km]

lrat      ratio of largest sub-area to total area

medrn      median annual rainfall [mm]

minel      elevation of catchment outlet above AHD [m]

msl      mainstream length to catchment boundary

nn      number of streams of order strm-1

pe      ratio of average annual rainfall to average class A pan evaporation

rlen      length of reaches modelled in RORB [km]

rlm      rlen/msl

rlt      rlen/tsl

rr      (mxel-minel)/msl

rrd      average annual rainfall/annual number of raindays

sa      number of sub areas in RORB

strm      Strahler stream order at the outlet

tsl      total stream length measured as total length of "blue lines" from a 1:100,000 scale map [km]

Note: A detailed outline of the parameters and the range of each used to derive the prediction equations is given in Dyer et al. (1994).

Table 7.2.3 Data used with Excel™ curve smoothing to draw graphs in Figure 7.2.1

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7							
-3.14	3.60	-3.14	3.70	-3.14	0	-3.14	0.40	-3.14	-2.50	-3.14	-2.00	-3.14	0.60
-3.00	2.90	-3.00	3.40	-2.75	0	-3.00	0.60	-3.00	-2.00	-3.00	-1.80	-3.00	0.40
-2.62	-0.20	-2.50	0.50	-2.25	0.40	-2.75	0.90	-2.50	0.50	-2.50	-0.50	-2.50	-0.70
-2.20	-2.40	-2.00	-1.10	-1.90	0.20	-2.25	0.60	-2.20	1.25	-2.10	0.50	-2.23	-1.10
-1.67	0.80	-1.75	-0.90	-1.50	0.40	-2.00	0.80	-1.75	0.50	-1.50	-1.50	-1.75	-0.50
-1.08	4.00	-1.32	-0.40	-1.00	1.00	-1.50	1.60	-1.25	-0.50	-1.00	-3.50	-1.25	0.30
-0.75	2.70	-0.75	-2.30	-0.50	1.50	-1.00	2.10	-0.75	1.25	-0.50	-1.80	-0.75	-0.60
-0.38	-1.00	-0.20	-4.30	0	0	-0.50	1.25	-0.13	3.50	0	1.60	-0.30	-1.50
0.20	-4.60	0.25	-2.80	0.50	-2.25	0	-0.30	0.50	1.50	0.35	2.50	0.25	0.10
0.75	-2.60	0.50	-1.00	0.80	-2.60	0.50	-0.90	1.00	-0.30	1.00	1.35	0.80	2.40
1.25	-1.30	1.00	1.50	1.25	-1.40	1.00	-1.10	1.25	-0.70	1.25	1.10	1.25	1.10
1.75	-1.30	1.50	0.20	1.50	-0.20	1.50	-1.60	1.75	-0.10	1.50	1.35	1.50	-0.40
2.25	0.85	2.00	-0.90	2.12	1.60	2.00	-2.20	2.00	-0.70	2.00	2.00	2.05	-2.30
2.62	3.00	2.50	1.50	2.50	1.00	2.50	-1.50	2.50	-2.50	2.50	0.60	2.50	-1.20
2.80	4.00	3.00	3.70	3.00	0.15	3.00	0	2.85	-3.20	3.00	-1.70	3.00	0.40
3.14	3.70	3.14	3.80	3.14	0	3.14	0.30	3.14	-2.80	3.14	-1.90	3.14	0.50

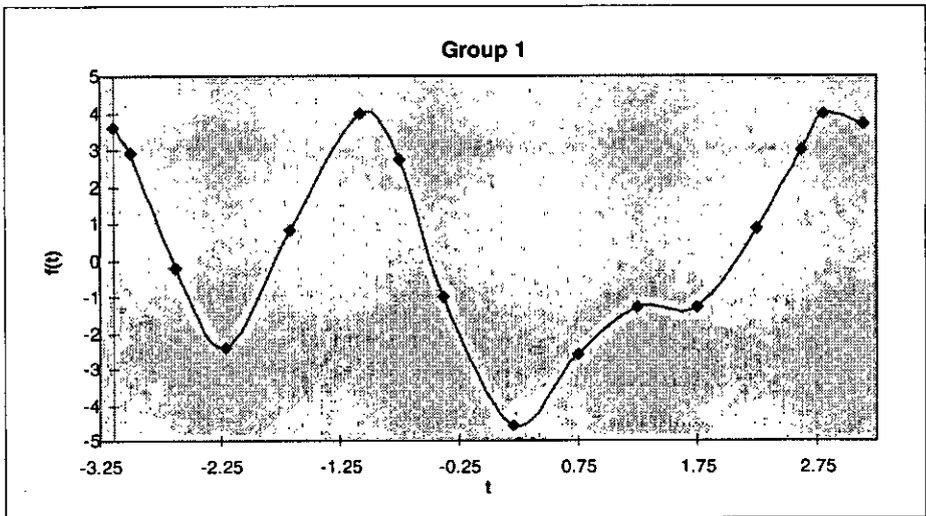


Figure 7.2.1 Andrews Curves for the 7 hydrologically similar groups

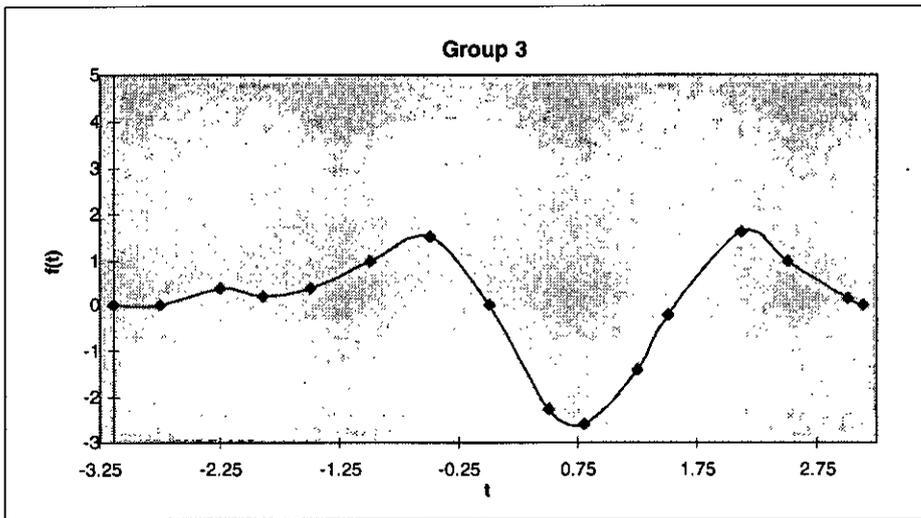
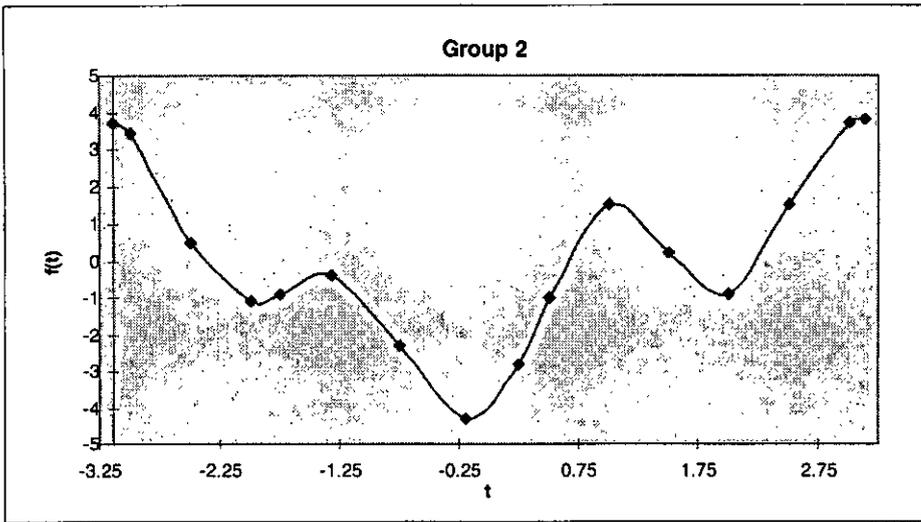


Figure 7.2.1(cont.) Andrews Curves for the 7 hydrologically similar groups

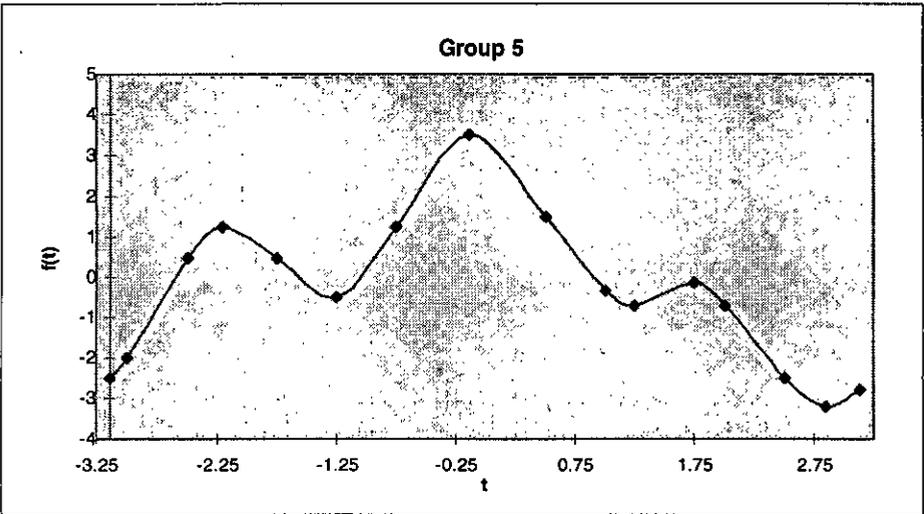
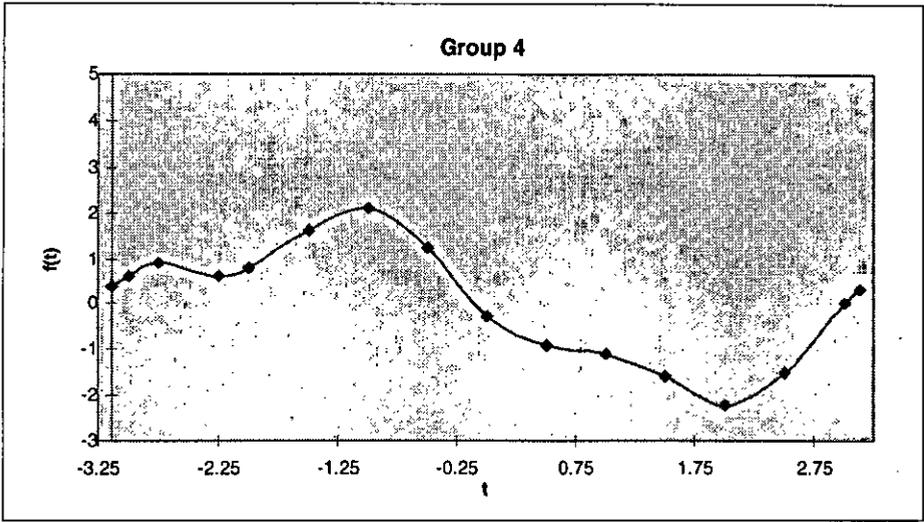


Figure 7.2.1 (cont.) Andrews Curves for the 7 hydrologically similar groups

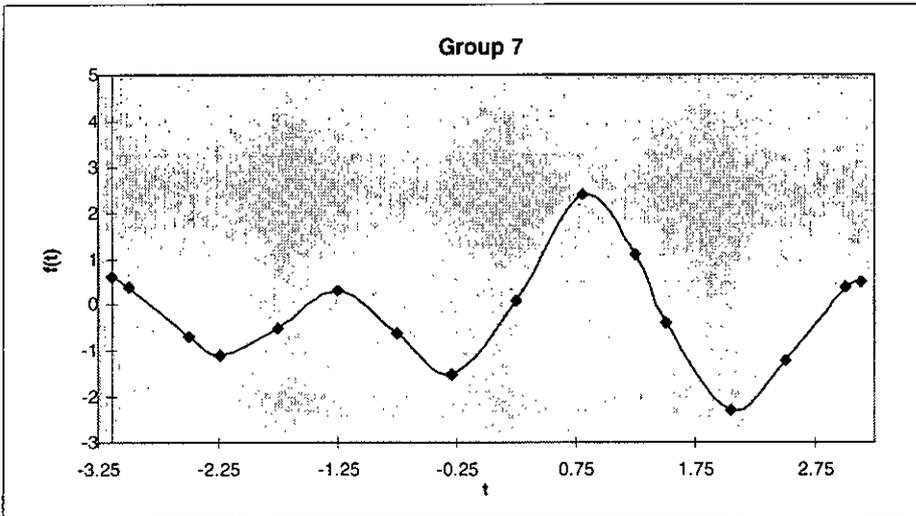
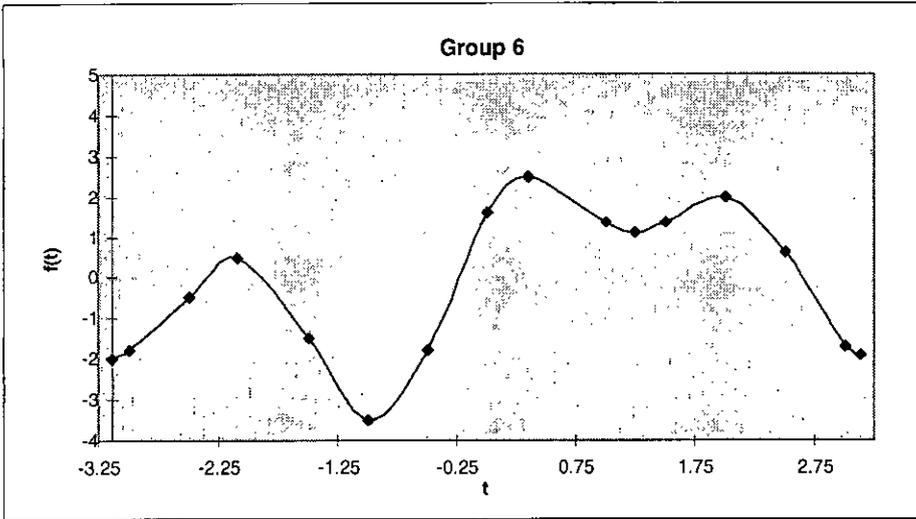


Figure 7.2.1 (cont.) Andrews Curves for the 7 hydrologically similar groups

**Example**

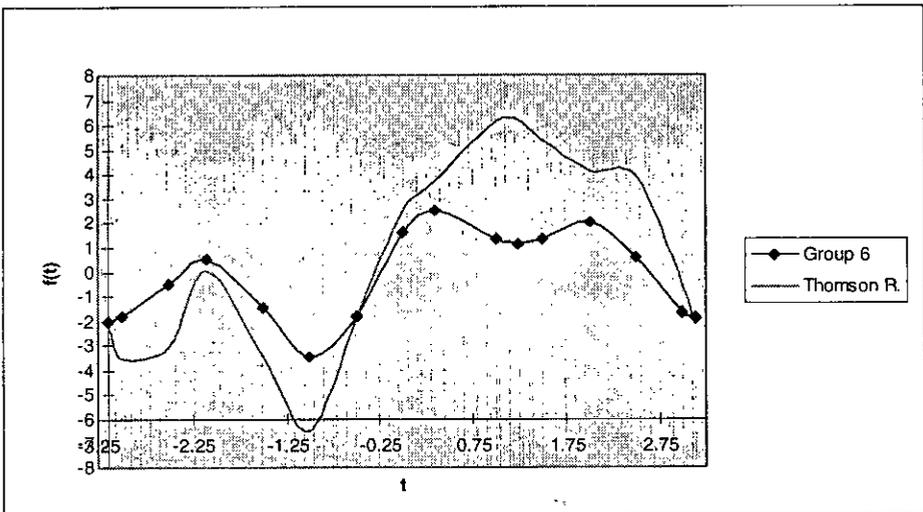
Compute an estimate of RORB  $k_C$  for the Thomson River at The Narrows given the following information:

longidig	146.4
d <sub>av</sub>	27.5
cd2	2
mxel	1500
pem	1348/551 = 2.45
for	1.00
rdy	160
rla	79.5/519 = 0.15

Substituting in Equations 7.2.1 - 7.2.8 gives the following:

x <sub>1</sub>	0.081
x <sub>2</sub>	4.87
x <sub>3</sub>	0.081
x <sub>4</sub>	1.13
x <sub>5</sub>	-0.556
x <sub>6</sub>	0.993
x <sub>7</sub>	1.55
x <sub>8</sub>	-1.38

These data are used to plot the Andrews Curve (Topic 7.1) and this is matched to determine the best fitting group, which in this case is Group 6 (Figure 7.2.2).



Hence Equation 6 in Table 7.2.2 is used to compute  $c_{0.8}$  :

$$c_{0.8} = 20.6 \text{ circ}^{0.79} \text{ for}^{1.38} \text{ rlt}^{-0.28} \text{ lnms}^{0.32}$$

Information is measured on 1:100,000 map sheets to give:

circ      area / (perimeter)<sup>2</sup> = 519/(124)<sup>2</sup> = 0.034

rlt      RORB stream length/total stream length = 79.5/606.5 = 0.131

lnms      ratio of stream length of order 5-1 / mainstream length = 24.5/53 = 0.462

Substituting gives  $c_{0.8} = 1.962 \Rightarrow k_C = 1.962 \times 27.5 = 54$

The value of  $k_C$  obtained by fitting RORB to flow data without baseflow was 50 so the estimation procedure is a reasonable one for this catchment.

### References

- Dyer, B.G., Nathan, R.J., McMahon, T.A. and I.C. O'Neill (1994): Development of regional prediction equations for the RORB runoff routing model. *CRC for Catchment Hydrology Report 94/1*.
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- Gordon, N.D., McMahon, T.A. and B.L. Finlayson (1992): *Stream Ecology - An Introduction for Ecologists*. John Wiley & Sons, 526pp.

## 7.3 PARAMETER ESTIMATION FOR THE MOSAZ MODEL

The MOSAZ model (Nathan et al., 1996) is a monthly yield model using daily rainfall and average monthly potential evapotranspiration data, plus relationships based upon catchment characteristics. MOSAZ is a two parameter model, with parameters being estimated from physical characteristics of the catchment. The two parameters are LS and KB. The model is available from the Water Bureau of the Department of Natural Resources and Environment, Victoria.

To date the model parameters have been determined for catchments in south-eastern Australia, and have been calculated for only a limited range of each of the characteristics.

### Physical Characteristics

To estimate the model parameters it is first necessary to ascertain the value of the following characteristics:

AREA	catchment AREA [km <sup>2</sup> ]
ELEV	elevation of the centroid of the catchment, in m above the Australian Height Datum (AHD) [m]
G2	fraction of the catchment underlain by sandstone.
RAIN	mean annual rainfall for the catchment [mm]
DENSITY	density of the catchment stream network, calculated by dividing the total length of the stream network (typically determined from measurement on NATMAP 1:100,000 topographic maps) by the catchment area [km]
LONGIT	longitude of the centroid of the catchment [degrees]

### Limits

The upper and lower limits of the physical characteristic values used in determination of the MOSAZ model parameters are given in the following table.

Variable	Minimum	Maximum
AREA (km <sup>2</sup> )	3.9	8400
ELEV (m)	30	1400
G2	0.00	1.00
RAIN (mm)	450	2300
LONGIT (°)	141.54	149.30
DENSITY (km <sup>-1</sup> )	0.52	1.42

If the catchment characteristics are outside these limits then careful attention should be given to deriving the streamflow feature of interest, including comparison of the flow statistics of the synthesised flow data from those with nearby catchments and/or with those from hydrologically similar catchments.

**BFI**

Once the physical characteristics have been determined, the Base Flow Index (BFI) should be calculated. Ideally this should be done from analysis of streamflow hydrographs using a technique such as the digital filtering approaches given in Topic 6.2. In the absence of such data, BFI can be calculated using Equation 7.3.1 from catchment characteristics already determined, along with the following:

ETPOT    mean annual potential evapotranspiration (mm). The valid range for ETPOT is 910 to 1200 mm.

FOREST    fraction of the catchment area covered by dense and medium forest. The valid range for FOREST is 0.00 to 1.00

LENGTH    length of mainstream [km]. The valid range for LENGTH is 1 to 280 km.

Note: In the following equations, a large number of significant figures are carried through. This should not be seen as a measure of accuracy of the final result!

$$\text{BFI} = 0.6168 + 0.000052 \text{ AREA} + 0.000192 \text{ ELEV} - .000448 \text{ ETPOT} + 0.1222 \text{ FOREST} - 0.125 \text{ G2} + 0.000233 \text{ RAIN} - 0.001268 \text{ LENGTH} \quad (7.3.1)$$

( $R^2 = 0.72$ ;  $s_e = 19\%$ ; number of samples,  $n = 164$ )

**MOSAZ Parameters**

The parameters LS and KB are determined through the following regression equations.

$$\text{LS} = -34.46 + 0.05519 \text{ ELEV} + 1424.106 \text{ BFI}^4 - 65.580 \text{ G2} + 0.1304 \text{ RAIN} \quad (7.3.2)$$

( $R^2 = 0.86$ ;  $s_e = 31\%$ ;  $n = 151$ )

$$1/\sqrt{\text{KB}} = 205.33 + 0.000919 \text{ AREA} - 35.596 \sqrt{\text{BFI}} - 3.0759 \text{ G2} + 0.00635 \text{ ELEV} - 1.7661 \text{ DENSITY} - 1.112 \text{ LONGIT} \quad 7.3.3)$$

( $R^2 = 0.69$ ;  $s_e = 18\%$ ;  $n = 86$ )

**Example**

Determine the value of the MOSAZ parameters LS and KB for Kinchington Creek at Ben Valley Road Bridge (402213). The characteristics for the catchment are as follows:

AREA (km <sup>2</sup> )	117.0	ELEV (m)	340
G2	0.00	RAIN (mm)	1000
LONGIT (°)	146.87	DENSITY (km <sup>-1</sup> )	1.93

Analysis of the streamflow record using method 3 in Topic 6.2 provides a BFI for the site of 0.42. Applying Equations 7.3.2 and 7.3.3 gives:

LS = 159

KB = 0.0032

(The software version of the MOSAZ estimation model gives 90 percentile values for these parameters of 50.0 and 267.0, and 0.001 and 0.0077, respectively.)

The parameters may now be used in the MOSAZ model along with daily rainfall and average potential evapotranspiration to give estimates of monthly yield.

### *References*

Nathan, R.J., Austin, K., Crawford, D. and N. Jayasuriya (1996): The estimation of monthly yield in ungauged catchments using a lumped conceptual model. *Australian Jour. of Water Resources*, In Press.

## 7.4 EMPIRICAL ESTIMATES OF FLOW PARAMETERS

A perennial problem in hydrology is the estimation of flow characteristics for catchments from which there is little or no streamflow data available. This topic presents some techniques suitable for such instances. It should be noted that these methods are based on limited data so estimates must be used with caution.

### *Flow frequency and flow duration characteristics from limited data*

These methods require some flow data to be measured at the site of interest in order to develop a basic relationship with a long term (index) station from a similar catchment. The methods are therefore useful for stations with short records or where it is feasible to measure flows for a few seasons. The steps for transposing frequency curves from the index station to the short term station are described in McMahon and Mein (1986) and summarised as follows:

- (i) For both the short term and index stations, construct a frequency curve (annual maximum, partial duration or low-flow frequency - see Gordon et al., 1992) for the period of **concurrent** record.
- (ii) From these curves, select discharges for several average recurrence intervals for both stations (e.g., 1.1, 1.5, 2, 5, 10, ... years). Plot discharges for the short term station against those of the index station in the log domain.
- (iii) Draw a curve through the points giving additional weight to the particular flow regime of interest (i.e., high or low). The curve will generally approach a straight line for higher values.
- (iv) Draw a frequency curve for the index station using the **full** record. Using this curve, choose several recurrence intervals and read off the discharge values. Use the relationship from (iii) to compute the equivalent flow for the site of interest.
- (v) A new frequency curve for the short term site is drawn through the translated points.

Flow duration curves can be derived in a similar way based on a series of concurrent measurements. The procedure is the same as above except that in step (ii), the discharges are selected for a number of percentage duration values (e.g., 10, 20, 30, ... 90).

## *Low Flow Characteristics*

Nathan and McMahon (1991) present a series of predictive equations for low flow characteristics based on measurable catchment characteristics from 184 rural catchments in Victoria and New South Wales (areas from 1 to 250 km<sup>2</sup>).

The flow characteristics considered are: baseflow index, mean annual flow, standard deviation of annual flows, river regime group (Haines et al., 1988), recession constant, flow duration curves, low flow frequency curves, spell duration, spell deficiency volumes, storage-yield analysis and SFB (Boughton, 1984) rainfall runoff model parameters.

The predictive equations are based on the following set of catchment characteristics measured from the 1:100,000 National Topographic Map Series and the 1:250,000 National Geologic Map Series: catchment area, latitude and longitude of catchment centroid, elevation of catchment centroid, fraction of forest cover, length of mainstream, slope of the central 75% of the mainstream length, stream frequency (number of junctions/catchment area), stream density, catchment shape (catchment perimeter/catchment area) and a series of geological indices representing the proportion of different rock types. Mean annual rainfall and coefficient of variation of monthly rainfall were also used.

Readers interested in these methods are directed to Nathan and McMahon (1991) for full descriptions and associated computer programs for undertaking the analysis. Nathan and Weinmann (1993) provide useful maps that may be used to transpose information between catchments.

## *Some General Relationships*

Table 7.4.1 presents equations for a number of key hydrological variables based on analysis of large data sets. The table includes the region from which the data were collated and the reference to the original work.

Table 7.4.1 Empirical equations for some key hydrological variables

EQUATION	STATISTICS	REGION	REFERENCE
$C_v = 2.0 \text{ MAR}^{-0.30}$	$n=974, r^2=37\%, S_E = +60\%, -38\%$	global	McMahon et al., 1992
$q_{\text{maf}} = 2.5 A^{0.63}$	$n=931, r^2=71\%, S_E = +208\%, -68\%$	global	McMahon et al., 1992
$q_{100}/q_{\text{maf}} = 5.8 C_{\text{vf}}^{0.92}$	$n=931, r^2=76\%, S_E = +38\%, -28\%$	global	McMahon et al., 1992
$\tau = [Z_p^2 C_v^2 / 4(1-D)] - dC_v^2$		global	McMahon and Mein, 1986; Gould, 1964
$\text{MAR} = 2300 A^{-0.35}$	$n=156, r^2=33\%, S_E = +157\%, -61\%$	Australia	McMahon, 1976
$C_v = 3.9 \text{ MAR}^{-0.33}$	$n=156, r^2=62\%, S_E = +35\%, -26\%$	Australia	McMahon, 1978
$q_{\text{maf}} = 3.3 A^{-0.44}$	$n=171, r^2=34\%, S_E = +286\%, -74\%$	Australia	McMahon, 1985
$q_{\text{smaf}} = 0.001 \text{ MAR}^{-1.02}$	$n=119, r^2=63\%, S_E = +210\%, -67\%$	Australia	McMahon, 1982
$\text{MAF} = 9.3 \times 10^{-6} A^{0.99} \text{ MAP}^{1.48}$	$n=80, r^2=97\%, S_E = +54\%, -35\%$	S.E. Australia	Gan et al., 1990
$C_v = 127 A^{0.06} \text{ MAP}^{-0.63} C_{\text{vp}}^{0.87}$	$n=81, r^2=56\%, S_E = +37\%, -27\%$	S.E. Australia	Gan et al., 1990
where			
A	catchment area [km <sup>2</sup> ]		
C <sub>v</sub>	coefficient of variation of annual flow		
C <sub>vf</sub>	coefficient of variation of annual peak flows		
C <sub>vp</sub>	coefficient of variation of annual precipitation		
D	draft from a reservoir as a ratio of annual flow		
d	factor to adjust annual flows to gamma distribution (see Table 5.3.1)		
MAF	mean annual flow [m <sup>3</sup> x 10 <sup>6</sup> ]		
MAP	mean annual precipitation [mm]		
MAR	mean annual runoff [mm]		
q <sub>maf</sub>	mean annual flood [m <sup>3</sup> s <sup>-1</sup> ]		
q <sub>smaf</sub>	specific mean annual flood [m <sup>3</sup> s <sup>-1</sup> km <sup>-2</sup> ]		
q <sub>100</sub>	100 year flood [m <sup>3</sup> s <sup>-1</sup> ]		
τ	required reservoir size as a ratio of mean annual streamflow		
Z <sub>p</sub>	standardised normal variate for p% risk of failure in any year (see Table 5.3.1)		

## References

- Boughton, W.C. (1984): A mathematical model for relating runoff to rainfall with daily data. *Civil Eng. Trans., Inst. Eng. Aust.*, **CE(8)**, pp. 83-97.
- Gan, K.C., McMahon, T.A. and I.C. O'Neill, (1990): *Errors in estimated streamflow parameters and storages for ungauged catchments*. *Water Resources Bull.*, **26**, pp. 443-450.
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- McMahon, T.A. (1976): Preliminary estimation of reservoir storage for Australian streams. *Civil Eng. Transactions, I.E. Aust.*, **CE18**, pp. 55-59.
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- McMahon, T.A. Finlayson, B.L., Srikanthan, R. and A.T. Haines (1992): *Global Runoff: Continental comparisons of annual flows and peak discharges*. Catena Verlag, Cremlingen-Destedt, West Germany, 166pp.
- Nathan R.J. and T.A. McMahon (1991): Estimating low flow characteristics in ungauged catchments. *Water Resource Management* **6(3)**, pp. 85-100.
- Nathan, R.J. and P.E. Weinmann (1993): *Low Flow Atlas for Victorian Streams*. Long Term Planning Guidelines Background Report - L3, Department of Conservation and Natural Resources, Victoria.

## 7.5 ESTIMATING RESERVOIR STORAGE VOLUME FOR UNGAUGED STREAMS

Estimation of storage for ungauged streams can be undertaken in a number of ways. Gan et al. (1988) reviewed four approaches based on data from catchments in south-east Australia. The method with the best predictive power was regression of the storage matrix wherein data from 71 catchments were used to derive storage sizes for a range of probabilities of failure and drafts using a behaviour analysis and lake evaporation formula. Regression equations were then derived for the relationship between storage size and a variety of physiographic and rainfall parameters. The resulting parameters for a predictive equation are presented in this topic. The storage values exclude dead storage.

### *Equation and parameters*

The equation used is of the following form:

$$\log_{10} S = a_0 + a_1 \log_{10} X_1 + a_2 \log_{10} X_2 + a_3 \log_{10} X_3 \dots\dots\dots (7.5.1)$$

where S is the storage in [million m<sup>3</sup>] and the variables X are draft (the amount of water required for release, given as a percentage of mean annual flow) [%], probability of failure (percentage of time the reservoir is empty) [%], catchment area [km<sup>2</sup>], mean annual rainfall [mm], coefficient of variation of monthly rainfall and coefficient of skew of monthly rainfall.

Note: This equation was derived for catchments ranging in area from 0.1 km<sup>2</sup> to 250 km<sup>2</sup> with no appreciable urban area and no large storages. It should be used only within these limits.

Table 7.5.1 Regression results (after Gan et al., 1988)

<b>Draft (%)</b>	<b>100-80</b>	<b>100-80</b>	<b>70-50</b>	<b>70-50</b>	<b>40-30</b>	<b>40-30</b>
<b>Prob. of failure</b>	<b>1-5</b>	<b>10-25</b>	<b>1-5</b>	<b>10-25</b>	<b>1-5</b>	<b>10-25</b>
Number of sites	639	852	639	852	426	568
R <sup>2</sup>	0.97	0.95	0.93	0.81	0.78	0.51
% Std. error	-36, 56	-44, 78	-51, 103	-70, 232	-70, 234	-85, 547

Table 7.5.2 Variables and their coefficients for each range of draft and probability of failure (after Gan et al., 1988)

Draft (%)	100-80	100-80	70-50	70-50	40-30	40-30
Prob. of failure	1-5	10-25	1-5	10-25	1-5	10-25
Draft(%)	3.3100	3.5129	2.4411	3.1886	2.3959	3.1058
Prob. of failure	-0.1462	-0.8038	-0.2292	-0.9779	-0.3754	-1.1857
Area [km <sup>2</sup> ]	1.1063	1.0890	1.0866	1.0196	0.9104	0.6689
Mean (ann.rainfall)	1.2170	1.2872	1.1609	0.9691		-0.8591
Cv (monthly rainfall)	1.1412	2.2367	1.9054	2.7287	1.4391	
CSkew(monthly rainfall)	0.0525	-0.1220	-0.1891	-0.4031	-0.3160	-0.5567
Constant (a <sub>0</sub> )	-10.5081	-10.2152	-8.4996	-8.3091	-4.7455	-2.6119

### Example

Estimate the storage size required for the Albert River at Hiawatha (227216) which has a catchment area of 31.1 km<sup>2</sup>, for a draft of 85% and a probability of failure of 5%. The mean annual rainfall is 1235 mm, the coefficient of variation and the coefficient of skew of monthly rainfall are 0.65 and 1.23 respectively.

Substituting into (7.5.1) gives:

$$\log_{10} S = -10.5081 + 3.31 \log_{10} 85 - 0.1462 \log_{10} 5 + 1.1063 \log_{10} 31.1 + 1.217 \log_{10} 1235 + 1.1412 \log_{10} 0.65 + 0.0525 \log_{10} 1.23$$

$S = 0.98$  million m<sup>3</sup> or approximately 1000 ML.

### References

- Gan, K.C., McMahon, T.A. and I.C. O'Neill (1988): Estimation of yields and storages for small rural ungauged catchments. *Conf. on Agr. Eng., I.E. Aust. Nat. Conf. Publ.* **88/12**, pp. 302-306.

## 7.6 ESTIMATING EXTREME FLOOD DISCHARGES

### *Regression equations for Probable Maximum Floods in South Eastern Australia*

Formal methods for the computation of the Probable Maximum Flood (PMF) are presented in ARR87 and must be followed for detailed studies. However, in many situations only an approximate value is needed and there is neither the time nor data available to follow those procedures. In such circumstances, quick, data frugal methods of PMF computation are obviously desirable.

This topic presents regression equations for the computation of the triangular PMF hydrograph. These were derived from analysis of PMF estimates from 56 catchments in South Eastern Australia (see Fig. 7.6.1) ranging in size from 1 km<sup>2</sup> to 10,000 km<sup>2</sup> (Nathan et al., 1994) and can be applied to catchments that do not have large lakes or artificial storages.

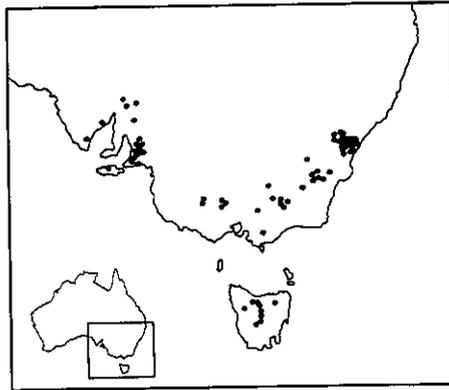


Figure 7.6.1 Geographical distribution of study dams (after Nathan et al., 1994)

$$Q_p = 129.1 A^{0.616} \quad [r^2=0.95, S_e = +36\% -26\%, n=56] \quad (7.6.1)$$

$$V = 497.7 A^{0.984} \quad [r^2=0.98, S_e = +39\% -28\%, n=56] \quad (7.6.2)$$

$$T_p = 1.062 \times 10^{-4} A^{-1.057} V^{1.446} \quad [r^2=0.89, S_e = +42\% -29\%, n=38] \quad (7.6.3)$$

$$T_r = \frac{V}{1.8Q_p} \quad (7.6.4)$$

where

- $Q_p$  peak flow [m<sup>3</sup>s<sup>-1</sup>]  
 $A$  catchment area [km<sup>2</sup>]  
 $V$  hydrograph volume [ML]  
 $T_p$  time to peak of the hydrograph [h]

$T_r$  length of hydrograph [h] derived by mass balance

For comparison, Figure 7.6.2 shows data from large measured floods in S. E. Australia along with PMF estimates and measured floods from around the world.

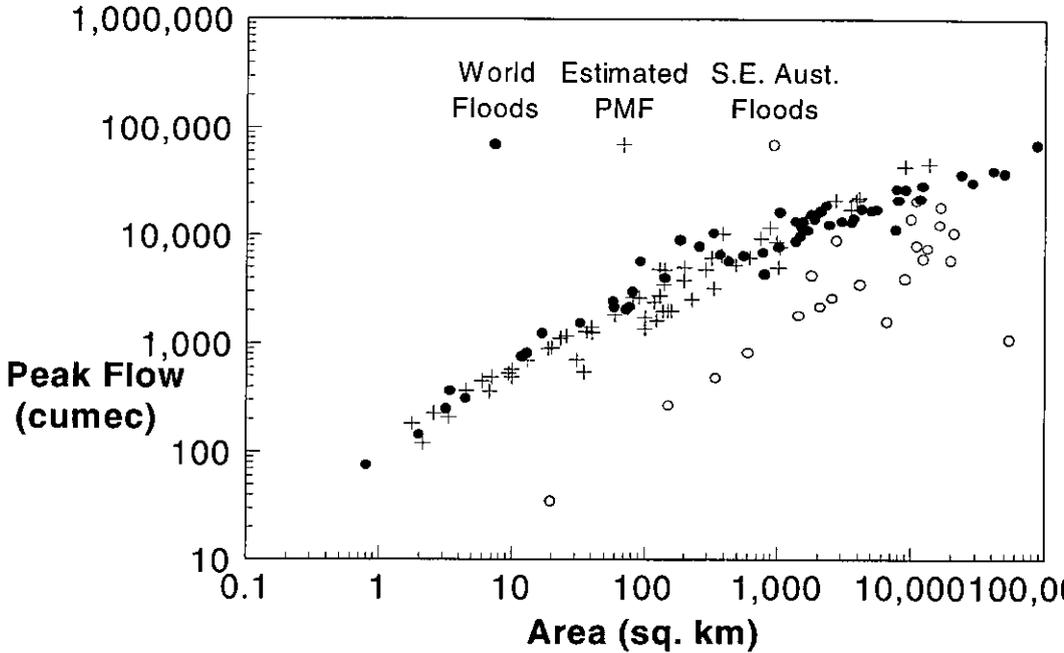


Figure 7.6.2. A comparison of Estimated PMFs with recorded floods in S.E. Australia and the world

*Regression equations for 1% flood flows near the Great Dividing Range in Victoria*

In general, the probabilistic rational method as described in ARR87 should be used to compute 1% annual exceedence probability (AEP) floods. As a quick computation of approximate magnitudes however, this section presents the results of a similar study to the one summarised above, where data were collated from flood studies on 105 sites either side of the Great Dividing range in Victoria (Nikolaou/von't Steen, DCNR, pers comm). Historical data from floods thought to be approaching or exceeding 1% events were also included. The resulting estimation equations for large floods, assumed to be **approximately** 1% AEP, are:

For rural catchments  $Q_{0.01} = 4.67 A^{0.763}$   $[r^2=.91, n=66]$  (7.6.5)

For urban catchments  $Q_{0.01} = 10.29 A^{0.71}$   $[r^2=.93, n=30]$  (7.6.6)

Note. These equations must not be applied to catchments that are affected by artificial or natural storages such as floodplains, reservoirs or breakaway channels.

### Example

Estimate the PMF and approximate 1% AEP flood flows for the Traralgon Creek at Traralgon South (226415) which has a catchment area = 128 km<sup>2</sup> and is rural.

From Equation 7.6.1  $Q_p = 129.1 (128)^{0.616} = 2564 \text{ m}^3\text{s}^{-1} \approx 2500 \text{ m}^3\text{s}^{-1}$

From Equation 7.6.2  $V = 497.7 (128)^{0.984} = 58950 \text{ ML} \approx 60000 \text{ ML}$

From Equation 7.6.3  $T_p = 1.062 \times 10^{-4} (128)^{-1.057} (58950)^{1.446} = 5.0 \text{ hours}$

From Equation 7.6.4  $T_r = 58950 / (1.8 \times 2564) = 12.8 \text{ hours} \approx 13 \text{ hrs}$

From Equation 7.6.5  $Q_{0.01} = 4.67 (128)^{0.763} = 189 \text{ m}^3\text{s}^{-1} \approx 200 \text{ m}^3\text{s}^{-1}$

Note: The ratio between PMF and 1% flow is larger than would be expected, indicating the limitations of using these simple regression approaches.

### *References*

Nathan, R.J., Weinmann, P.E. and S.A. Gato (1994): A quick method for estimating the probable maximum flood in South Eastern Australia. *Water Down Under 94. Hydrology and Water Resources Symposium I.E. Aust. Nat. Conf. Publ. 94/10*, pp.229-234.

### *Other Reading*

Rodier, J.A. and M. Roche (1984): World catalogue of maximum observed floods. *IAHS Publ. No. 143*.



## 8.1 ESTIMATING RIVER CONDITION BY SURVEY

### *Environmental Rating*

An estimation of the environmental condition of a stream or river reach is sometimes required. Approaches to environmental rating contain a degree of subjectivity, particularly in what is or is not included, what one is measuring against, and the temporal and spatial resolution. Because of this, it is important that a particular method is designed to achieve clearly defined objectives, that the audience for the outcomes is identified and that the group collecting the data are considered.

The assessment of stream reaches in Victoria (Mitchell, 1990) provides one technique for obtaining an environmental rating of a river reach. Like most such methods, there are a great many underlying assumptions that may render the method inappropriate for a particular application. The procedure attempts to provide an absolute measure of environmental condition rather than a measure of condition that recognises the natural characteristics of the site. The procedure rates environmental condition against an ideal that may or may not be appropriate to the reach under investigation and it would be possible for a site in pristine condition to rate poorly if it did not conform to the assumed ideal of environmental condition. This limits the geographical flexibility of the technique.

Potential users are advised to consider carefully whether the method is appropriate for their intended use. The further reading list contains references to other methods, including a method for Queensland streams using an expanded version of the method presented here (Jackson and Anderson, 1994) and a more comprehensive index for Victoria that is under development (Ladson et al., 1996).

In the approach presented here, streams are categorised according to catchment area, as follows:

- *minor* streams being those less than 5000 ha
- *tributary* streams being between 5000 and 30,000 ha, and
- *major* streams being greater than 30,000 ha.

The data categories used by Mitchell (1990) and reproduced below were developed for the "State of the streams survey" by Ian Drummond and Associates (1985). Mitchell (1990) applied the environmental ratings to these categories.

Table 8.1.1 Environmental Rating

Stream Size	Environmental Rating				
	Very Poor	Poor	Moderate	Good	Excellent
<b>Bed Composition</b>					
Minor	All sand	Gravel/ sand	Gravel; some sand; some cobbles	At least 10% cobbles; mainly shingle	Boulders/ cobbles shingles; small amount of gravel or finer
Tributary	N/A	All sand	Gravel/ sand	Mainly shingle; gravel	Shingle; cobble; gravel
Major	N/A	N/A	All sand	Shingle; gravel; sand	Shingle; cobbles present
<b>Proportion of Pools and Riffles</b>					
Minor	100% pool or riffle	90% pool or riffle	70-80% pool or riffle	60% pool or riffle	50% pool or riffle
Tributary	Intermittent pools	All pools	< 10% riffles	10-30% riffles	>30% riffles
Major (Only 3 rating categories)	Intermittent pools or very shallow	N/A	100% pools	N/A	Some riffles
<b>Bank Vegetation</b>					
All	Introduced ground cover with lots of bare ground; occasional tree	Introduced ground cover; little native overstorey or understorey or predominantly exotic cover	Moderate cover; mixed natives/ exotics; or one side cleared and other side undisturbed	Minor clearing	Mainly undisturbed native vegetation
<b>Verge Vegetation</b>					
All	Bare or Pasture	Very narrow corridor of native vegetation or exotics	Wide corridor of mixed natives or exotics; or one side cleared and other native and wide	Mainly undisturbed native; <30 m or some exotics or reduced cover of natives	Mainly undisturbed native vegetation; > 30 wide
<b>Cover for Fish</b>					
All	None	Poor	Moderate	Good	Abundant
<b>Average Flow Velocity</b>					
Minor	0	0.1 - 0.2 ms <sup>-1</sup>	0.3 - 0.6 ms <sup>-1</sup>	0.6 - 0.7 ms <sup>-1</sup>	> 0.8 ms <sup>-1</sup>
Tributary	0	0.1 - 0.2 ms <sup>-1</sup>	0.3 - 0.6 ms <sup>-1</sup>	0.6 - 0.7 ms <sup>-1</sup>	> 0.8 ms <sup>-1</sup>
Major	N/A	0	0.1 ms <sup>-1</sup> (pools)	0.2 ms <sup>-1</sup> (pools)	> 0.3 ms <sup>-1</sup> (pools)
<b>Water Depth</b>					
Minor	Dry or trickle	< 0.2 m	0.3 - 0.5 m	0.6 - 1.0 m	> 1.0 m
Tributary	Dry or trickle	< 0.2 m	0.3 - 0.5 m	0.6 - 1.0 m	> 1.0 m
Major	< 0.3 m	0.4 m	0.5 - 0.9 m	1.0 - 2.0 m	> 2.0 m
<b>Underwater Vegetation</b>					
All	0 or > 80% cover	1 - 5% or 60 - 80% cover	5 - 20% cover	20 - 30% cover	30 - 60% cover
<b>Organic Debris</b>					
All	0	0 - 10% cover	10 - 20% cover	20 - 40% cover	40% cover
<b>Erosion/ Sedimentation</b>					
All	Extensive	Significant	Moderate; affecting parts of reaches	Only spot erosion	Stable; no erosion or sedimentation

In assessing a reach it is necessary to consider a reasonable length, such as the shorter of 2.5 times the stream meander wavelength or 25 times the average stream width.

An overall environmental rating on the scale “very poor; poor; moderate; good; excellent” is obtained from consideration of the 10 factors listed above. Factors are weighted according to their significance for biological diversity and productivity at the site, with highest weighting given to bed composition, fish cover and bank and verge vegetation.

### Example

Assess the environmental rating of the Aberfeldy River downstream of Beardmore. The catchment area is larger than 30,000 ha, so can be considered a major stream.

Factor	Site Details	Rating (Table 8.1.1)
Bed composition	Shingle, with cobbles	Excellent
Proportion of pools and riffles	Some riffles	Excellent
Bank vegetation	Undisturbed native forest	Excellent
Verge vegetation	Undisturbed native forest	Excellent
Cover for fish	Abundant	Excellent
Average flow velocity	0.35 ms <sup>-1</sup>	Excellent
Water depth	1.4 m	Good
Underwater vegetation	20 - 30 % cover	Good
Organic debris	20 - 40% cover	Good
Erosion/ sedimentation	Stable	Excellent

Overall rating of the stream reach: Excellent

### *References*

- Ian Drummond and Assoc. Pty. Ltd. (1985): *Statewide Assessment of physical stream conditions - Phase 1*. Department of Water Resources, Victoria.
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- Ladson, A. R., Doolan, J., White, L., Metzeling, L. and D. Robinson (1996): *Index of stream condition as a tool to aid management of rivers. Hydrology and Water Resources Symposium, I.E. Aust. Natl. Conf. Publ., 96/05*, pp. 325-331.
- Mitchell, P. (1990): *The Environmental Condition of Victorian Streams*. Water Victoria. Department of Water Resources, Victoria.

### *Further Reading*

- Tilleard, J.W. (1986): A statewide assessment of physical stream conditions in Victoria. *Hydrology and Water Resources Symposium. I.E. Aust. Natl. Conf. Publ., 86/13*, pp. 380-381.

## 8.2 SIGNIFICANCE OF 'BANKFULL' DISCHARGE AND METHODS FOR ITS ESTIMATION

### *Background*

The size and shape of alluvial channels is determined by a combination of discharge, sediment load, sediment particle size, roughness, velocity, slope and characteristics of the bank such as particle size and vegetative cover. Of these, discharge and sediment load are generally the most important in determining channel form. It is convenient for both analytical and conceptual purposes to think of a particular discharge as a "dominant" or "channel-forming" discharge (Gordon et al., 1992; Harrelson et al., 1994). This single value is used to represent the range of discharges that primarily govern channel size and shape. It is common to estimate such a value from the "bankfull" discharge because it is easier to measure than some other "channel forming" discharges and relates to the most efficient channel operation (i.e. flow resistance is lowest). It should be noted that there is a range of definitions for "channel-forming" discharges and not all of these relate to bankfull conditions (Knighton, 1984).

Conceptually, bankfull discharge is easy to define as the discharge at which a channel just begins to overflow onto the floodplain. Operationally, the definition is not so clear since banks are not always the same height on both sides of the river, and in upland streams, there may be no obvious transition to a floodplain (see Williams, 1978; Woodyer, 1968; Riley, 1972). Harrelson et al. (1994) provide guidelines to the establishment of bankfull discharge in difficult circumstances, and these are summarised below.

It is suggested that a range of methods be used at as many sites as possible in order to obtain a realistic, averaged value of bankfull level. This will need to be converted to discharge using a rating curve (if the site is near a gauging station) or the slope/area method based on Manning's equation or similar (see below). At gauging sites, the bankfull discharge can often be determined from a break-point in the rating curve.

**POINT BARS** can be used to fix a minimum value on the bankfull level. The highest elevation of the point bar (deposited material on the inside of meander bends) is measured since it is the location where the floodplain is still being constructed by deposition

**CHANGES IN VEGETATION** can provide a useful estimate of bankfull level. Look for the lower limit of perennial vegetation or sharp changes in vegetation density or type.

**CHANGES IN CROSS-SECTIONAL SLOPE** (e.g. near vertical to horizontal) are often the most obvious indicators in lowland streams. Care is needed where there are multiple terraces present. In these cases, the vegetation and bench stability need to be considered.

In steep channels, **BANK UNDERCUTS** often define close to "bankfull" level (perhaps slightly underestimating the true value)

### *Recurrence interval of bankfull discharge*

There is no standard recurrence interval for bankfull discharge. A wide range of values has been suggested and most are between the 1:1 year and 1:2 year events. These estimates are generally determined from the annual flood series so recurrence intervals of less than 1 year are not observed by this method. If these analyses were undertaken on the partial series (i.e. a flood series above a given magnitude from throughout a year), quite different recurrence intervals would result. Because of these problems, bankfull discharge should never be defined on the basis of a predetermined recurrence interval.

Once a bankfull discharge has been determined, it may be of interest to compute the recurrence interval of that flow. For all recurrence intervals less than 10 years, the partial series must be used.

### *Slope-Area method for converting bankfull level to discharge*

This technique is fully described in Gordon et al. (1992). It consists of a series of measurements taken in a reach of river where uniform flow conditions exist in order to calculate discharge using an equation such as Manning's:

$$Q = \frac{1}{n} AR^{\frac{2}{3}} S^{\frac{1}{2}} \quad (8.2.1)$$

where:

- Q discharge [ $\text{m}^3\text{s}^{-1}$ ]  
 n Manning's coefficient  
 A cross sectional area of flow [ $\text{m}^2$ ]  
 R hydraulic radius [m] =  $\frac{\text{cross sectional area of flow}}{\text{wetted perimeter}}$   
 S slope of the energy gradient - approximated by slope of water surface [m/m]

- (i) Choose a straight reach of length at least five times the width, with constant slope and where the water surface slope is parallel to the bed slope. It is important that the velocity is approximately the same at each cross section.
- (ii) Survey at least three cross sections to compute average hydraulic radius (R) and flow area (A) as well as water surface slope. If water surface slope cannot be measured, average bed slope may be used. This is used to approximate the energy gradient S in Equation 8.2.1.
- (iii) Estimate a value for Manning's n and compute the discharge. Manning's n may be estimated from Table 8.2.1 or from the references listed under further reading at the end of this topic.

Note: There are many sources of error in this method. Choice of a suitable reach is vital to ensure uniform flow and to minimise variation in R, A and S. In addition, the choice of Manning's  $n$  is highly subjective. By far the best way to determine bankfull discharge is to actually measure it in the field or if the site is near a gauging station, use the gauging record to determine the bankfull discharge from bankfull level.

Table 8.2.1 Manning's  $n$  values for streams of top width < 30 m after Chow (1959)

Description of Channel	Minimum	Normal	Maximum
(a) Clean, straight, no deep pools	0.025	0.030	0.033
(b) Same as (a), but more stones and weeds	0.030	0.035	0.040
(c) Clean, winding, some pools and shoals	0.033	0.040	0.045
(d) Same as (c), but some weeds and stones	0.035	0.045	0.050
(e) Same as (c), at lower stages, with rougher slopes	0.045	0.050	0.060
(f) Same as (d) but more stones	0.045	0.050	0.060
(g) Sluggish reaches, weedy, deep pools	0.050	0.070	0.080
(h) very weedy reaches, deep pools or floodways with heavy stand of timber and scrub	0.075	0.100	0.150
Mountain streams (no vegetation in channel, banks steep, trees and scrub on banks submerged at high stages)			
(a) Streambed of gravel, cobbles, few boulders	0.030	0.040	0.050
(b) Bed is cobbles with large boulders	0.040	0.050	0.070

## References

- Gordon, N.D., McMahon, T.A. and B.L. Finlayson (1992): *Stream Ecology - An Introduction for Ecologists*. John Wiley & Sons, 526pp.
- Harrelson, C.C., C. L. Rawlins and J.P. Potyondy (1994): *Stream Channel Reference Sites: An illustrated guide to field technique*. US.Dept. of Agr. Forest Service General Technical Report **RM-245**.
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### *Further Reading*

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Nunally, N.R. (1967): Definitions and identification of channel overbank deposits and their respective roles in flood-plain formation. *Professional Geographer*, **19**, pp. 1-4.

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## 8.3 THRESHOLD OF RIVER BED MOVEMENT

The conditions under which stream-bed material will be mobilised can be investigated using approaches based upon critical velocity or critical shear stress (Gordon et al.; 1992). Most estimation approaches are based upon experimental values using uniform grain sizes, so are only indicative of the values in the field. Gordon et al. (1992) provide original references and discussion of issues related to sediment movement in streams. This topic presents two methods for calculating sediment movement criteria. One based on stream velocity and the other on bed shear stress.

### *Critical Velocity*

Critical velocity,  $V_C$ , is the velocity above which particles of a given size are found to be generally transported. The graph below, based upon the Hjulstrom curves, relates mean velocity to average particle diameter,  $d$ , with critical velocity indicated by the lower dashed line. The erosion / transportation transition falls between the two upper dotted lines. This approach has generally been replaced by shear stress methods which have been found to give more consistent results.

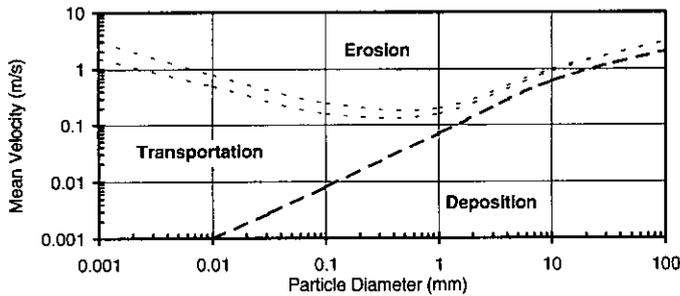


Figure 6.3.1 Sediment mobility based on Hjulstrom curve

Alternatively, the US Bureau of Reclamation approach relates  $V_C$  to particle diameter.

$$V_C = 0.155 \sqrt{d} \quad (8.3.1)$$

### **Relative Bed Stability (RBS)**

RBS is one measure of the stability of the stream bed, calculated as the ratio of  $V_C$  to the bed velocity,  $V_B$ . Values of greater than 1 indicate stable conditions.

$$RBS = V_C / V_B \quad (8.3.2)$$

Bed velocity has been estimated from  $0.7V$ , with the average stream velocity,  $V$ , calculated as the average of the velocity measured at  $0.2h$  ( $V_{0.2h}$ ) and  $0.8h$  ( $V_{0.8h}$ ), where  $h$  is stream depth. This is approximate only and more complex methods of estimating  $V_B$  may be necessary for rough beds.

## Bed Shear Stress

The shear stress,  $\tau$  [ $\text{Nm}^{-2}$ ], at the stream bed is calculated from consideration of fluid and stream properties. Shear stress methods have generally replaced methods based on critical velocity.

$$\tau = \rho g R S \quad (8.3.3)$$

where

$\rho$	water density, 1000 [ $\text{kgm}^{-3}$ ]
$g$	gravitational acceleration, 9.81 [ $\text{ms}^{-2}$ ]
$R$	hydraulic radius = the stream cross-sectional area / the wetted perimeter, [m]
$S$	channel energy line slope, approximated by the water surface slope for uniform flow [m/m]

### Critical Shear Stress

Critical shear stress for stream bed particles,  $\tau_c$ , is calculated using (8.3.4).

$$\tau_c = \theta_c g d (\rho_s - \rho) \quad (8.3.4)$$

where

$\theta_c$	dimensionless critical shear stress
$\rho_s$	sediment density, often approximated as 2650 [ $\text{kgm}^{-3}$ ]

Shields curve (Figure 8.3.2) allows estimation of  $\theta_c$  as a function of the Grain Reynolds Number,  $R_{e*}$ .

Grain Reynolds Number is calculated from (8.3.5).

$$R_{e*} = V_* k / \nu \quad (8.3.5)$$

where

$k$  effective roughness height, approximated by the 85th or 50th percentile bed particle diameter,  $d_{85}$  or  $d_{50}$

$V_*$  shear stress velocity, calculated through one of the following.

$$V/V_* = 5.75 \log(h/k) + 6 \quad (8.3.6)$$

$$V_{0.2h}/V_* = 5.75 \log(0.2h/k) + 8.5 \quad (8.3.7)$$

$$V_{0.8h}/V_* = 5.75 \log(0.8h/k) + 8.5 \quad (8.3.8)$$

If (8.3.6), (8.3.7), and (8.3.8) yield similar results, then use  $V_*$  based upon (8.3.6). Otherwise use the average of the three values.

$\nu$  kinematic viscosity, calculated as a function of temperature,  $T$  [ $^{\circ}\text{C}$ ], by

$$\nu = 0.0018 / (1000 * (1 + 0.0337T + .00022T^2)) \quad (8.3.9)$$

The values represented by the Shields curve were based upon experiments with a large amount of scatter. The value of 0.06 in the hydraulically rough region is based on non-cohesive particles larger than 6 mm. The indicative values of  $\theta_c$  for different stream bed conditions, given in Table 8.3.1, indicate the range of values that occur.

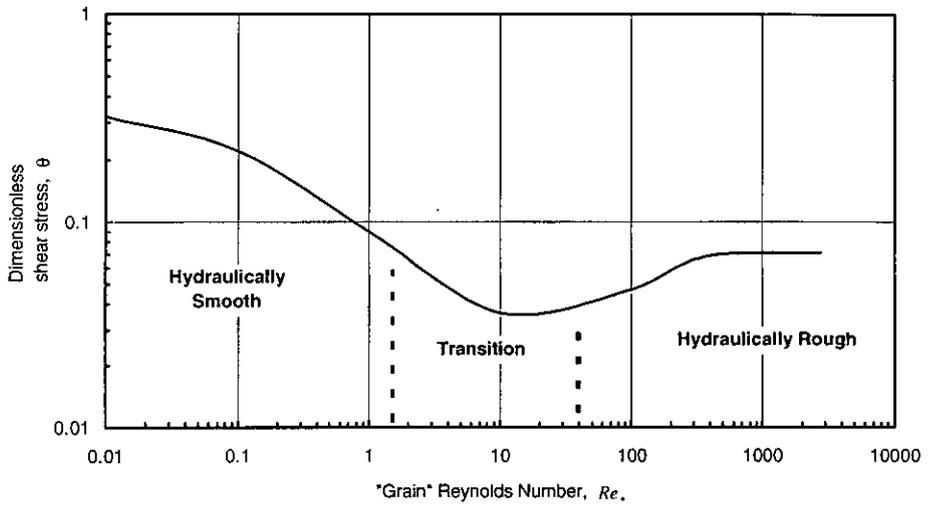


Figure 8.3.2 Shields Curve

Table 8.3.1 Indicative values of  $\theta_c$  for different stream bed conditions

Stream Bed Condition	$\theta_c$
Loose sand and gravels with large voids	0.01 - 0.035
Uniform materials or well settled beds	0.035 - 0.065
Close packed bed, with smaller particles in voids	0.065 - 0.10
Highly imbricated beds	> 0.10

**Example**

Determine the particle size that will be entrained for the Acheron River ( $R = 0.4$ ,  $S = 0.012$ ,  $\rho = 1000$ ,  $\rho_s = 2650$ ,  $\theta_c = 0.060$  i.e. fully rough) using the critical shear stress method

From (6.3.3), 
$$\tau = 1000(9.81)(0.40)(0.012) = 47 \text{ Nm}^{-2}$$

Then, from (6.3.4) 
$$\tau_c = 47 = 0.06(9.81)(d)(2650-1000)$$

giving 
$$d = 0.048 \text{ m} = 48 \text{ mm}$$

**Note that for fully rough flow when  $\theta_c = 0.060$ ,  $\tau_c$  [Nm<sup>-2</sup>] approximately equals particle size in mm.**

*References*

Gordon, N.D., McMahon, T.A. and B.L. Finlayson (1992): *Stream Hydrology - An Introduction for Ecologists*. John Wiley & Sons, 526pp.

## 8.4 SEDIMENT TRANSPORT

It is convenient to consider transported sediment as originating from the channel itself and from the catchment which then enters the channel. Total sediment transport is the sum of these two. The portion originating from the catchment is virtually always "supply limited", meaning that the amount of sediment transported is determined by the availability of material rather than the capacity of the flow to move the material. This means that equations cannot be derived just from flow characteristics. Stream channels, however, are often composed of loosely packed sands and gravels which are not cohesive and it is possible, in principle, to establish empirical relations between transported load and flow characteristics. The equations given in this topic are for transport of non-cohesive channel material. The first three are for "bed-load" i.e., material moving along and near the surface of the bed and generally not entrained in the flow proper. The fourth is a "total load" formula that is intended to represent all material moving down a stream.

Formulae for computing sediment transport are many and varied. This topic gives three examples but readers are referred to the references and further reading lists at the end of the topic for a broader range of methods. The one feature that applies to all of them, however, is that they are accurate only under the limited conditions for which they were derived. Because these conditions are generally so restrictive, they must be used in other cases and so the expected level of accuracy is poor. It is therefore necessary when using an equation, to be fully aware of the conditions under which it was derived and any assumptions that may limit its use. In the early 1970s, the American Society of Civil Engineers undertook a review of formulae and their reports form an excellent introduction to the range of approaches available (ASCE, 1971; Vanoni, 1975). The methods described in this topic are at the simpler end of the spectrum of available approaches.

### *Du Boys Bed Load Formula (eg. Raudkivi, 1990)*

This is a very early sediment transport equation based on the concept of excess shear stress and has a basic form that is common to many more recent equations. It is still widely used since it is easy to apply and is no poorer than many other equations. As presented here, it is suitable for sediments in the size range 0.25 mm to 4 mm in diameter.

$$q_s = C_s \tau_0 (\tau_0 - \tau_c) \quad (8.4.1)$$

$C_s$  and  $\tau_c$  can be related to  $d_{50}$  [mm] by:

$$C_s = \frac{7.011 \times 10^{-6}}{d_{50}^{0.75}} \quad (8.4.2)$$

$$\tau_c = 0.5985 + 0.9097 d_{50} \quad (8.4.3)$$

where

- $q_s$  volumetric sediment discharge per unit width [ $m^3 s^{-1} m^{-1}$ ]  
 $\tau_0$  shear stress for the conditions being considered [ $Nm^{-2}$ ] see Topic 8.3.  
 $d_{50}$  median size of bed sediment [mm]

**It must be noted that the value of  $\tau_c$  is NOT the same as the value given in the Shields Curve (see Topic 8.3). It was derived much earlier than Sheild's work and Equation 8.4.3 must be used when substituted into Equation 8.4.1.**

*Meyer - Peter Bed Load Formula (Meyer-Peter and Muller, 1948)*

This equation was derived from laboratory studies using well-sorted river sediments in the size range 3.1 mm to 28.6 mm. Because of the flow conditions in the flumes, rough bed forms did not develop so flow resistance was due to grain roughness not bed-form roughness. The formula should therefore be used only where the resistance due to bed-forms is low and the bed particle size is relatively coarse.

$$g_s^{\frac{2}{3}} = 250g^{\frac{2}{3}}S - 42.5d_{50} \quad (8.4.4)$$

where

- $g_s$  sediment discharge per unit width [ $kg s^{-1} m^{-1}$ ]  
 $q$  water discharge per unit width [ $m^3 s^{-1} m^{-1}$ ]  
 $S$  slope of stream [ $m m^{-1}$ ]  
 $d_{50}$  median size of bed sediment [m]

*Engelund-Hansen Bed Load Formula (Engelund and Hansen, 1967)*

This formula was also based on flume experiments, but in this case with smaller particle sizes in the range 0.19 mm to 0.93 mm. Its authors do not recommend it for cases where the median particle diameter is less than 0.15 mm, when the geometric standard deviation of grain sizes is greater than two, or where ripples rather than dunes are present in the bed forms.

$$g_s = 0.05\rho_s V^2 \sqrt{\frac{d_{50}}{g\left(\frac{\rho_s}{\rho} - 1\right)}} \left(\frac{\tau_0}{gd_{50}(\rho_s - \rho)}\right)^{\frac{3}{2}} \quad (8.4.5)$$

where

$\rho_s$	density of the sediment grains [ $\text{kgm}^{-3}$ ]
$\rho$	density of water [ $\text{kgm}^{-3}$ ]
$V$	mean flow velocity [ $\text{ms}^{-1}$ ]
$g$	acceleration due to gravity = $9.81$ [ $\text{ms}^{-2}$ ]
$\tau_0$	shear stress = $\rho gR S$ , where $R$ is the hydraulic radius [ $\text{m}$ ]
other variables are as defined previously	

### *Ackers and White (White, 1972; Ackers and White, 1973)*

The Ackers and White method is intended to represent total load and was derived from a relatively wide range of data. It is claimed that the method should give "good results" defined as 50% or more of the results having:

$$\frac{1}{2} < \left( \frac{\text{estimated } g_s}{\text{measured } g_s} \right) < 2 \quad (8.4.6)$$

The equations are based on 3 dimensionless quantities,  $G_{gr}$ ,  $F_{gr}$  and  $D_{gr}$  related to transport and stream power; shear stress and immersed weight; and immersed weight and viscous forces respectively.

$$G_{gr} = \frac{q_s D_m}{q d_{50}} \left[ \frac{u^*}{V} \right]^n = C \left[ \frac{F_{gr}}{A} - 1 \right]^m \quad (8.4.7)$$

$$F_{gr} = \frac{u^{*n}}{\sqrt{g d_{50} \left[ \frac{\rho_s}{\rho} - 1 \right]}} \left( \frac{V}{\sqrt{32} \log \left( \frac{10 D_m}{d_{50}} \right)} \right)^{1-n} \quad (8.4.8)$$

$$D_{gr} = d_{50} \left( \frac{g \left[ \frac{\rho_s}{\rho} - 1 \right]}{v^2} \right)^{\frac{1}{3}} \quad (8.4.9)$$

$$\text{for } D_{gr} > 60; \quad n = 0, m = 1.5, A = 0.17, C = 0.025 \quad (8.4.10)$$

$$\text{for } 1 < D_{gr} < 60; \quad n = 1 - 0.56 \log(D_{gr}) \quad (8.4.11)$$

$$m = 1.34 + \frac{9.66}{D_{gr}} \quad (8.4.12)$$

$$A = 0.14 + \frac{0.23}{\sqrt{D_{gr}}} \quad (8.4.13)$$

$$\log C = 2.86 \log D_{gr} - (\log D_{gr})^2 - 3.53 \quad (8.4.14)$$

where all variables are as defined previously and

- $q_s$  volumetric sediment transport rate [ $m^3 s^{-1}$ ]
- $D_m$  mean flow depth [m]
- $u^*$  =  $(gRS)^{0.5}$
- $\rho_s$  density of sediment [ $kg m^{-3}$ ]
- $\rho$  density of water [ $kg m^{-3}$ ]
- $\nu$  kinematic viscosity [ $m^2 s^{-1}$ ]

**Example**

Compute using each method, the calculated sediment discharge rate from a river which has the following characteristics: width = 10m, depth = 5m (rectangular cross-section), slope=0.00033, a constant discharge =  $87 m^3 s^{-1}$ . The sediments have  $d_{50} = 0.3mm$  and a density of  $2650 kgm^{-3}$ . The average stream temperature is  $15^{\circ}C$  giving a kinematic viscosity of approximately  $1.1 \times 10^{-6} m^2 s^{-1}$  and a density of  $999.1 kgm^{-3}$ .

1. From Equation 8.4.2  $C_s = \frac{7.011 \times 10^{-6}}{0.3^{0.75}} = 0.0000172$

from Equation 8.4.3  $\tau_c = 0.5985 + 0.9097 \times 0.3 = 0.8714$

from Topic 8.3,  $\tau = \rho g R S = 999.1 \times 9.81 \times (50/20) \times 0.00033 = 8.086$

from Equation 8.4.2  $q_s = 0.0000172 \times 8.086 (8.086 - 0.8714) = 0.0010 m^3 s^{-1} m^{-1}$

for a width of 10m and a sediment density of  $2650 kgm^{-3}$ , total bed load discharge  
 $= 0.0010 \times 2650 \times 10 = 26.5 \approx 27 kgs^{-1}$

2. Meyer-Peter equation is not appropriate because the sediment is too small.

2. Mean flow velocity =  $87/50 = 1.74 ms^{-1}$ ,  $R = (5 \times 10)/(10+5+5) = 2.5m$ ,  $\gamma = 1.0$ ,  $\gamma_s = 2.65$

substituting into Equation 8.4.5 gives:

$$g_s = 0.05 \times 2650 \times (1.74)^2 \times \{0.0003/[9.81 \times (2.65-1)]\}^{0.5} \times \{[(999 \times 9.81 \times 2.5 \times 0.00033)/[9.81 \times 0.0003(2650-999)]]^{1.5}$$

$$g_s = 3.7 kgs^{-1} m^{-1}$$

for a width of 10 m, total bed load discharge = **37 kgs<sup>-1</sup>**

$$3. u^* = (9.81 \times 2.5 \times 0.00033)^{0.5} = 0.090 \text{ ms}^{-1}$$

substituting into Equation 8.4.9 gives:

$$D_{gr} = 0.0003[(9.81 \times (2.65-1)/(1.1 \times 10^{-6})^2]^{0.33333} = 7.12, \text{ therefore use equations (6.4.11 -6.4.14) to calculate n, m, A, C giving: } n = 0.5226; m = 2.697; A = 0.2262; C = 0.01518$$

substituting into (6.4.8) gives:

$$F_{gr} = [(0.090)^{0.5226}]/[9.81 \times 0.0003(2.65-1)]^{0.5} \times \{1.74/[5.657 \log(10 \times 5/0.0003)]\}^{1-0.5226}$$

$$F_{gr} = 1.055$$

substituting into Equation 8.4.7 gives:

$$G_{gr} = (q_s \times 5)/(8.7 \times 0.0003) \times (0.90/1.74)^{0.5226} = 0.01518[(1.055/0.2262)-1]^{2.697}$$

$$q_s = 0.00124 \text{ m}^3 \text{ s}^{-1} \text{ m}^{-1}$$

For a width of 10 m and density of  $2650 \text{ kgm}^{-3}$ , total sediment discharge

$$= 10 \times 2650 \times 0.00124 = 33 \text{ kgs}^{-1}$$

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